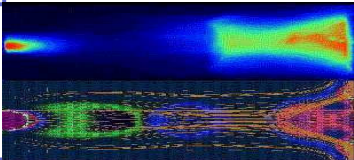


Dr Jasmin Smajic
ABB Corporate Research
Baden - Dättwil
jasmin.smajic@ch.abb.com



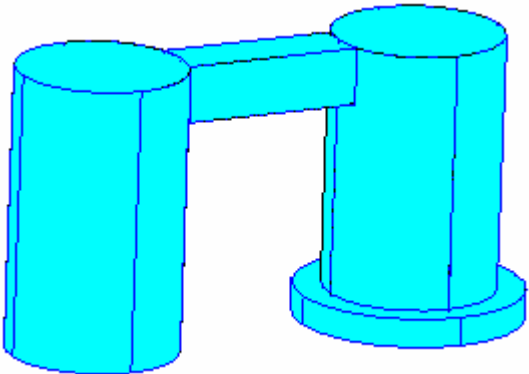
Introduction to FEM



Outline

- Introduction to the basic idea of FEM
- **Nodal triangular element**
- 2D electromagnetic analysis
 - Laplace-Poisson (static field computation)
 - Helmholtz equation (wave propagation and scattering)
 - Eigenvalue problems (resonators, waveguides)
- **Vector triangular element**
 - Inhomogeneous waveguide (hybrid modes analysis)
- **Nodal tetrahedral element**
- 3D electromagnetic analysis
 - Laplace-Poisson (static field computation)
- **Vector tetrahedral element**
 - Helmholtz equation (3D analysis of the photonic crystal waveguides and waveguide discontinuities, radiation losses analysis)
- Conclusions

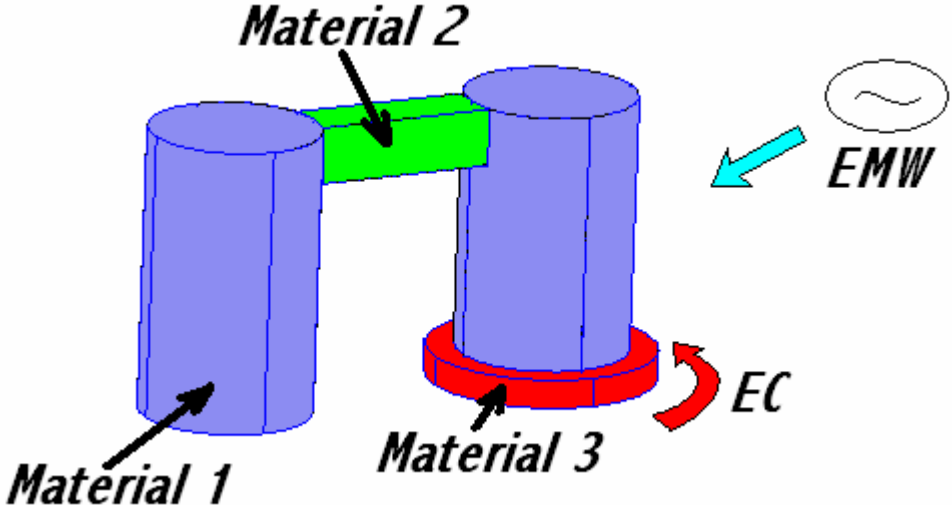
Introduction to the basic idea of FEM



Mathematical description

$$D\vec{f}(\vec{x}) = 0, \vec{x} \in \Omega$$

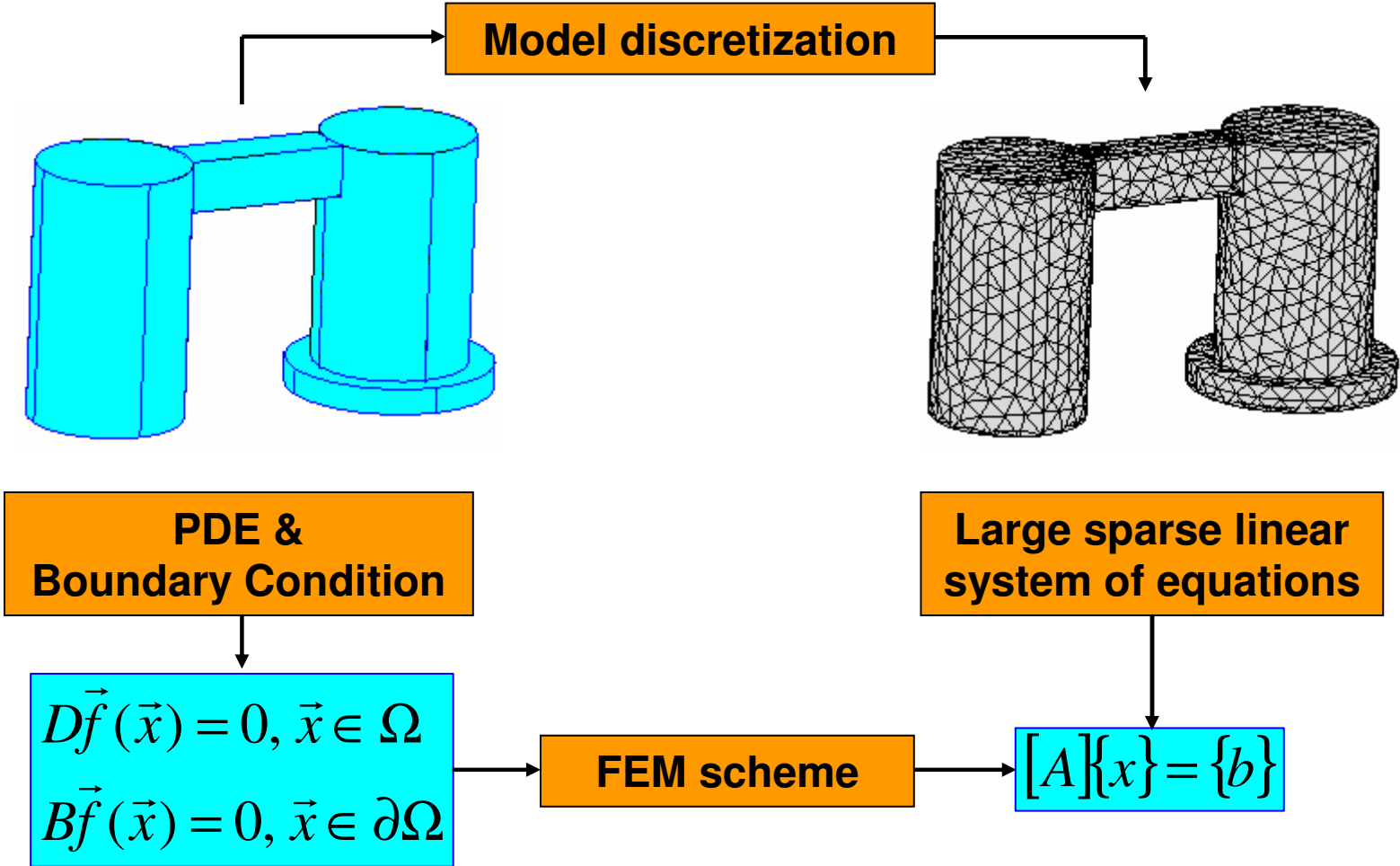
$$B\vec{f}(\vec{x}) = 0, \vec{x} \in \partial\Omega$$



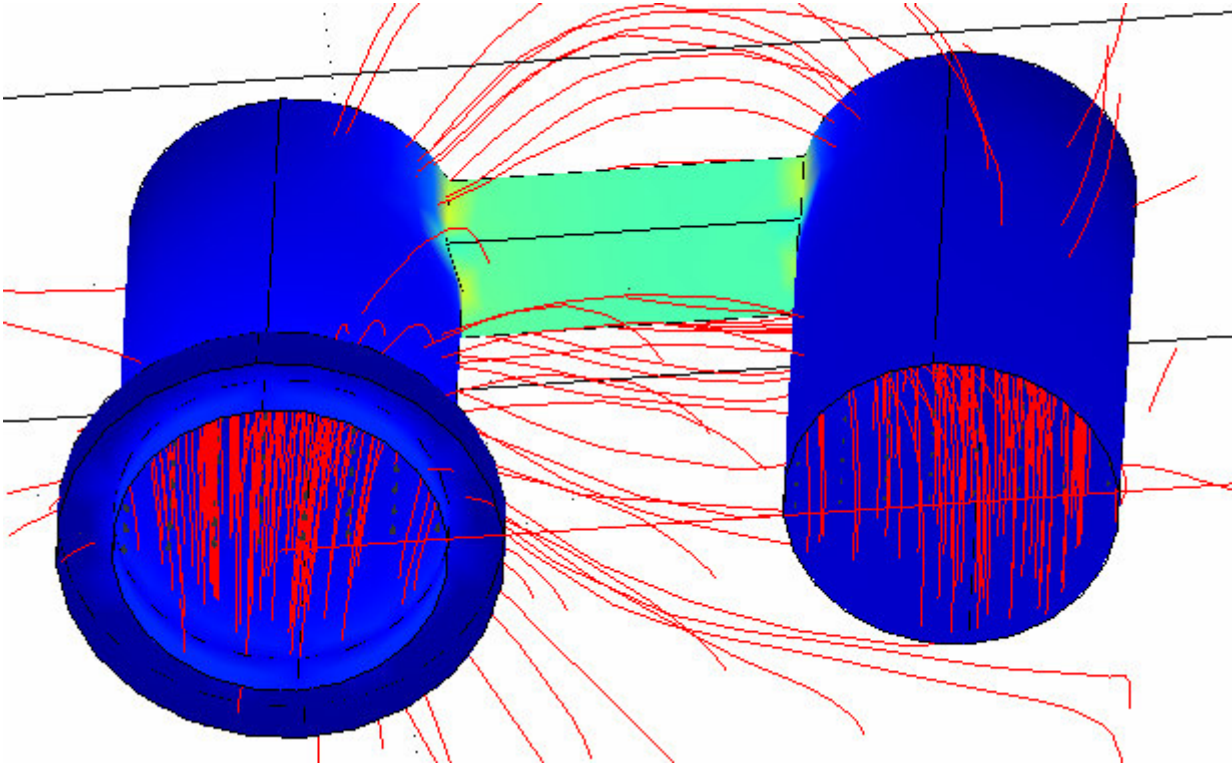
Model parameters

material: (μ, σ, ϵ)
 frequency: (f)
 sources: $(\vec{E}_s, \vec{H}_s, \vec{J}_s)$

Introduction to the basic idea of FEM



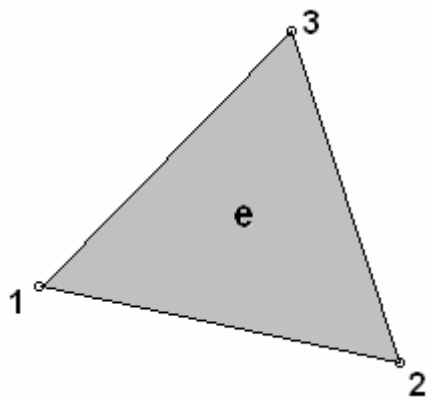
Introduction to the basic idea of FEM



“Fancy” pictures as a final result of field calculation

- Magnetic, electric field, potential...
- Electromagnetic energy, force ...
- Temperature, pressure, velocity...

Nodal triangular element



Linear triangular nodal element

$$\Phi^e(x,y) = a^e + b^e x + c^e y$$

$$\Phi^e(x,y) = \sum_{j=1}^3 N_j^e(x,y) \cdot \Phi_j^e$$

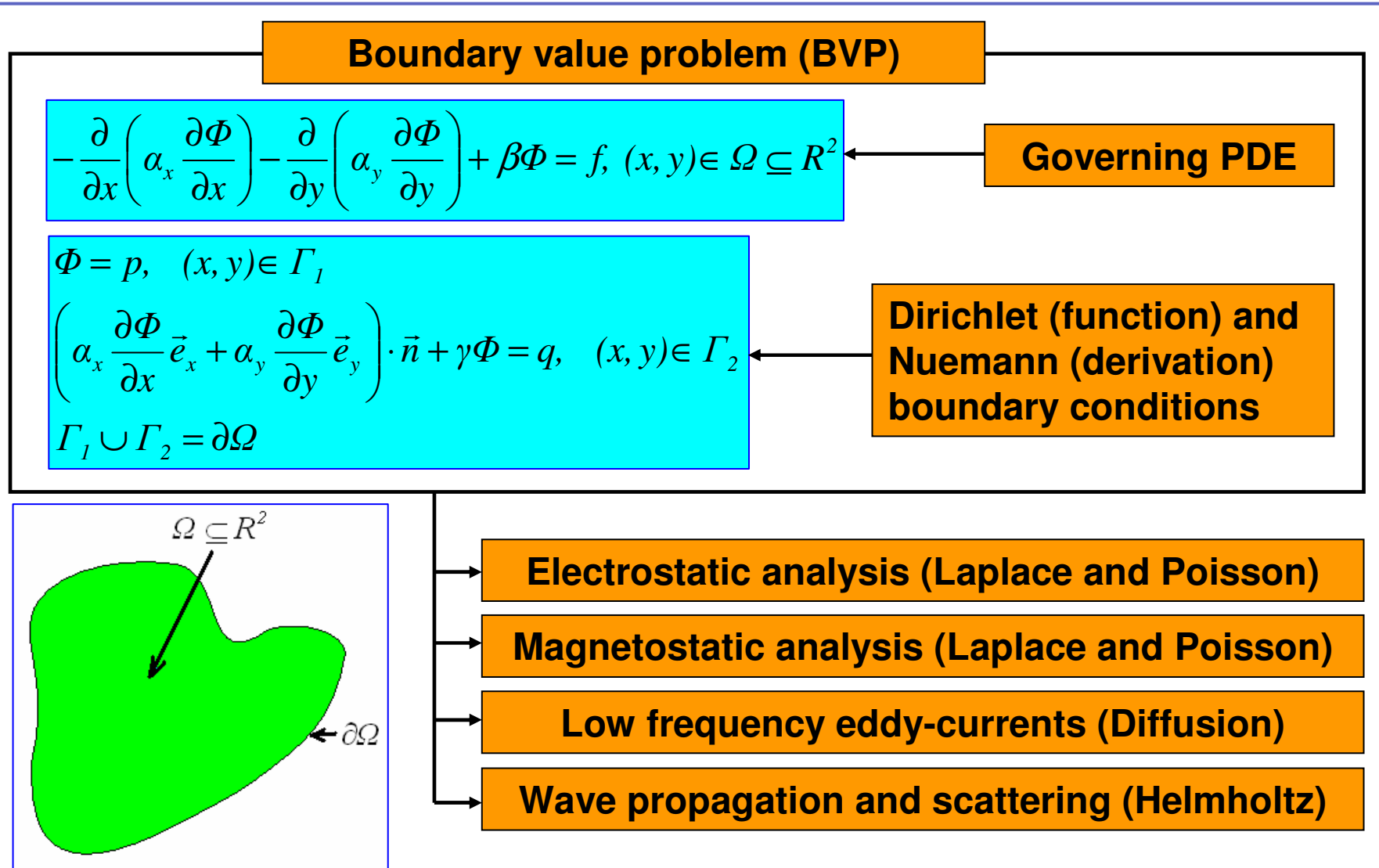
$$N_j^e(x,y) = \frac{1}{2\Delta^e} (a_j^e + x b_j^e + y c_j^e), \quad j = 1, 2, 3$$

$$a_1^e = x_2^e \cdot y_3^e - x_3^e \cdot y_2^e \quad b_1^e = y_2^e - y_3^e \quad c_1^e = x_3^e - x_2^e$$

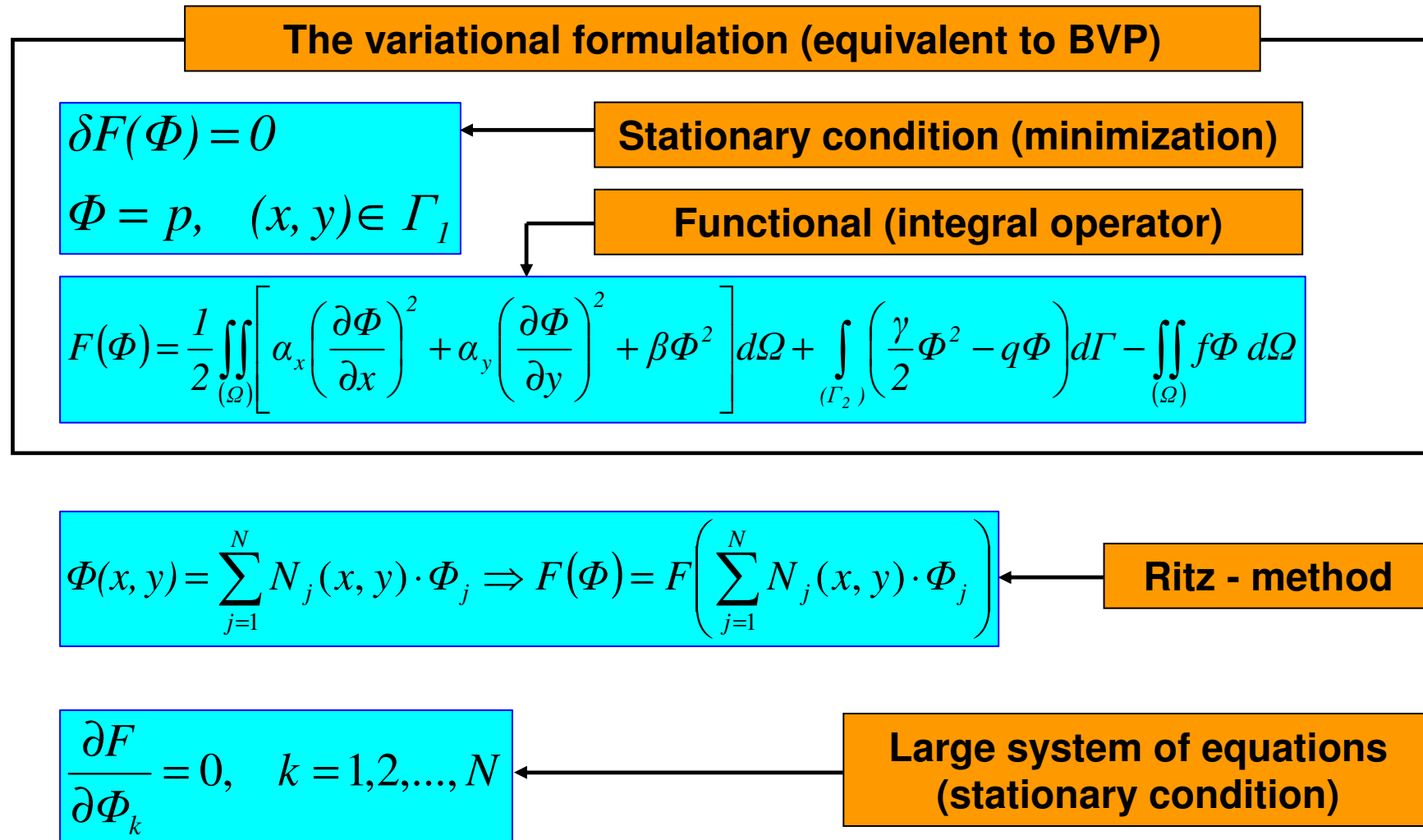
$$a_2^e = x_3^e \cdot y_1^e - x_1^e \cdot y_3^e \quad b_2^e = y_3^e - y_1^e \quad c_2^e = x_1^e - x_3^e$$

$$a_3^e = x_1^e \cdot y_2^e - x_2^e \cdot y_1^e \quad b_3^e = y_1^e - y_2^e \quad c_3^e = x_2^e - x_1^e$$

2D electromagnetic analysis



2D electromagnetic analysis



2D electromagnetic analysis

The finite element method

Local character of shape functions

$$N_i(x, y) = 0, \quad (x, y) \in \Omega - \Omega_i, \quad \Omega_i = \bigcup_{e \in \Omega^e} \Omega^e$$

$$\Phi^e(x, y) = \sum_{j=1}^3 N_j^e(x, y) \cdot \Phi_j^e$$

Concept of elemental contribution

$$F(\Phi) = \sum_{e=1}^{N_e} F^e(\Phi^e) \rightarrow \left\{ \frac{\partial F}{\partial \Phi} \right\} = \sum_{e=1}^{N_e} \left\{ \frac{\partial F^e}{\partial \Phi^e} \right\} = \sum_{e=1}^{N_e} ([K^e] \{\Phi^e\} - \{b^e\}) = 0$$

$$[K] \{\Phi\} = \{b\}$$

2D electromagnetic analysis

Concept of elemental contribution

$$F(\Phi) = \sum_{e=1}^{N_e} F^e(\Phi^e) \quad \left\{ \frac{\partial F}{\partial \Phi} \right\} = \sum_{e=1}^{N_e} \left\{ \frac{\partial F^e}{\partial \Phi^e} \right\} = \sum_{e=1}^{N_e} ([K^e] \{\Phi^e\} - \{b^e\}) = 0 \quad [K] \{\Phi\} = \{b\}$$

$$\left\{ \frac{\partial F^e}{\partial \Phi^e} \right\} = [K^e] \cdot \{\Phi^e\} - \{b^e\}$$

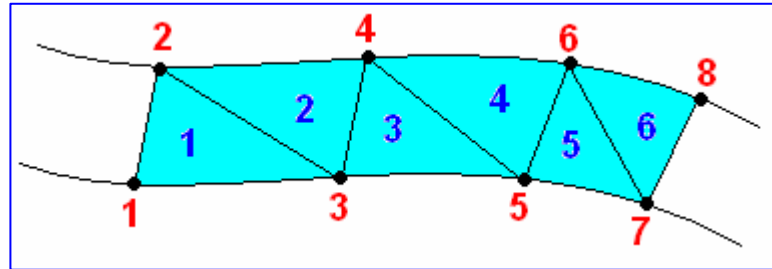
$$\left\{ \frac{\partial F^e}{\partial \Phi^e} \right\} = \left\{ \frac{\partial F^e}{\partial \Phi_1^e} \quad \frac{\partial F^e}{\partial \Phi_2^e} \quad \frac{\partial F^e}{\partial \Phi_3^e} \right\}^T \quad \{\Phi^e\} = \{\Phi_1^e \quad \Phi_2^e \quad \Phi_3^e\}^T$$

$$K_{ij}^e = \iint_{(\Omega^e)} \left(\alpha_x \frac{\partial N_i^e}{\partial x} \frac{\partial N_j^e}{\partial x} + \alpha_y \frac{\partial N_i^e}{\partial y} \frac{\partial N_j^e}{\partial y} + \beta N_i^e N_j^e \right) d\Omega, \quad i, j = 1, 2, 3$$

$$b_i^e = \iint_{(\Omega^e)} f N_i^e d\Omega, \quad i = 1, 2, 3$$

2D electromagnetic analysis

Concept of elemental contribution (linear nodal element)



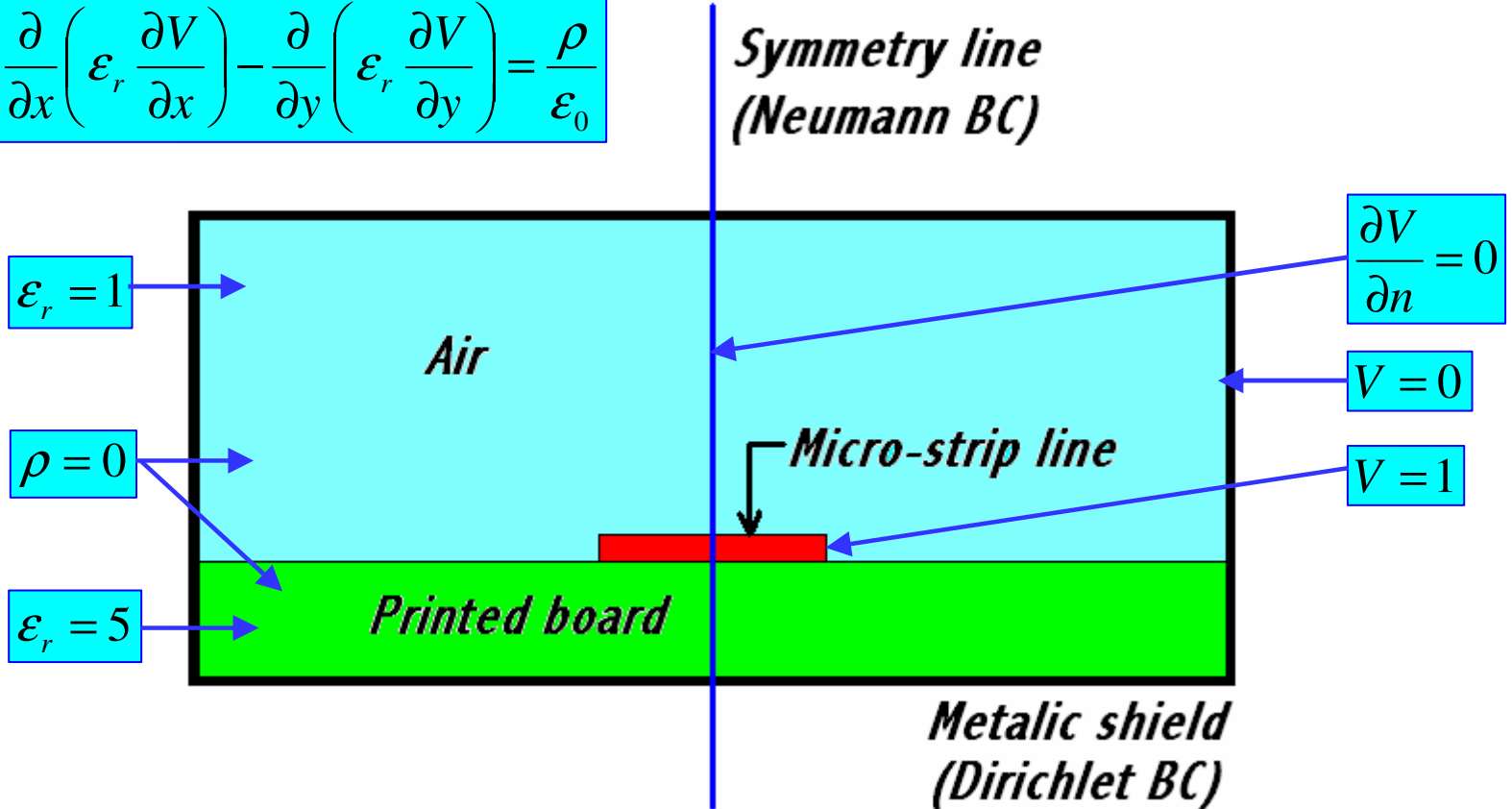
$[K^e]$ $e = 4$	K_{11}	K_{12}	K_{13}						
	K_{21}	K_{22}	K_{23}	K_{24}					
	K_{31}	K_{32}	K_{33}	K_{42}	K_{35}				
	K_{41}	K_{42}	K_{43}	K_{44}	K_{45}	K_{46}			
			K_{53}	K_{54}	K_{55}	K_{56}	K_{57}		
				K_{64}	K_{65}	K_{66}	K_{67}	K_{68}	
					K_{75}	K_{76}	K_{77}	K_{78}	
						K_{86}	K_{87}	K_{88}	

$\{b^e\}$ $e = 4$	b_1
	b_2
	b_3
	b_4
	b_5
	b_6
	b_7
	b_8

2D electromagnetic analysis

Laplace equation - electrostatic example

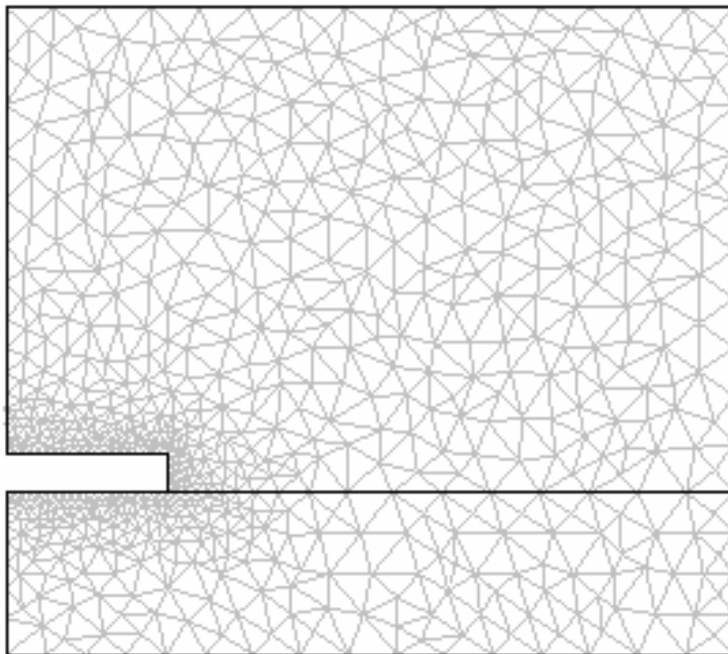
$$-\frac{\partial}{\partial x} \left(\epsilon_r \frac{\partial V}{\partial x} \right) - \frac{\partial}{\partial y} \left(\epsilon_r \frac{\partial V}{\partial y} \right) = \frac{\rho}{\epsilon_0}$$



2D electromagnetic analysis

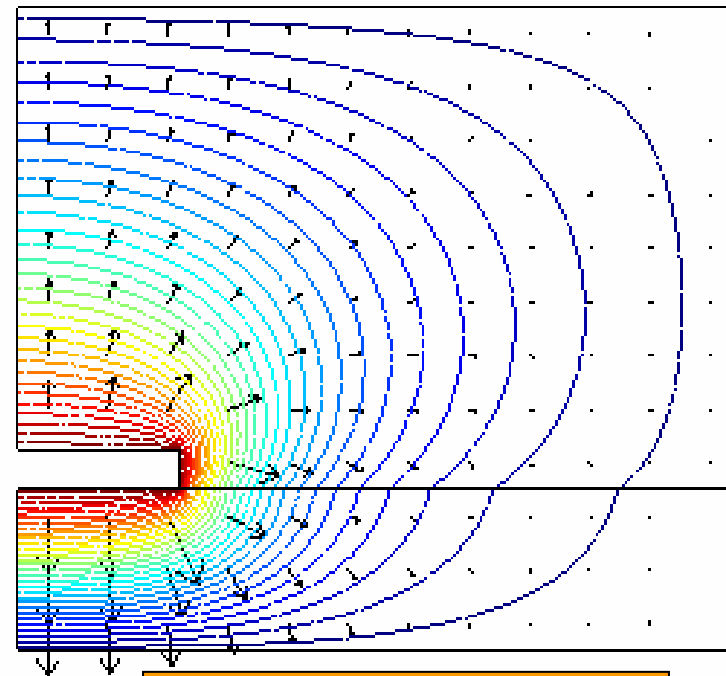
Laplace equation - electrostatic example

Triangular mesh



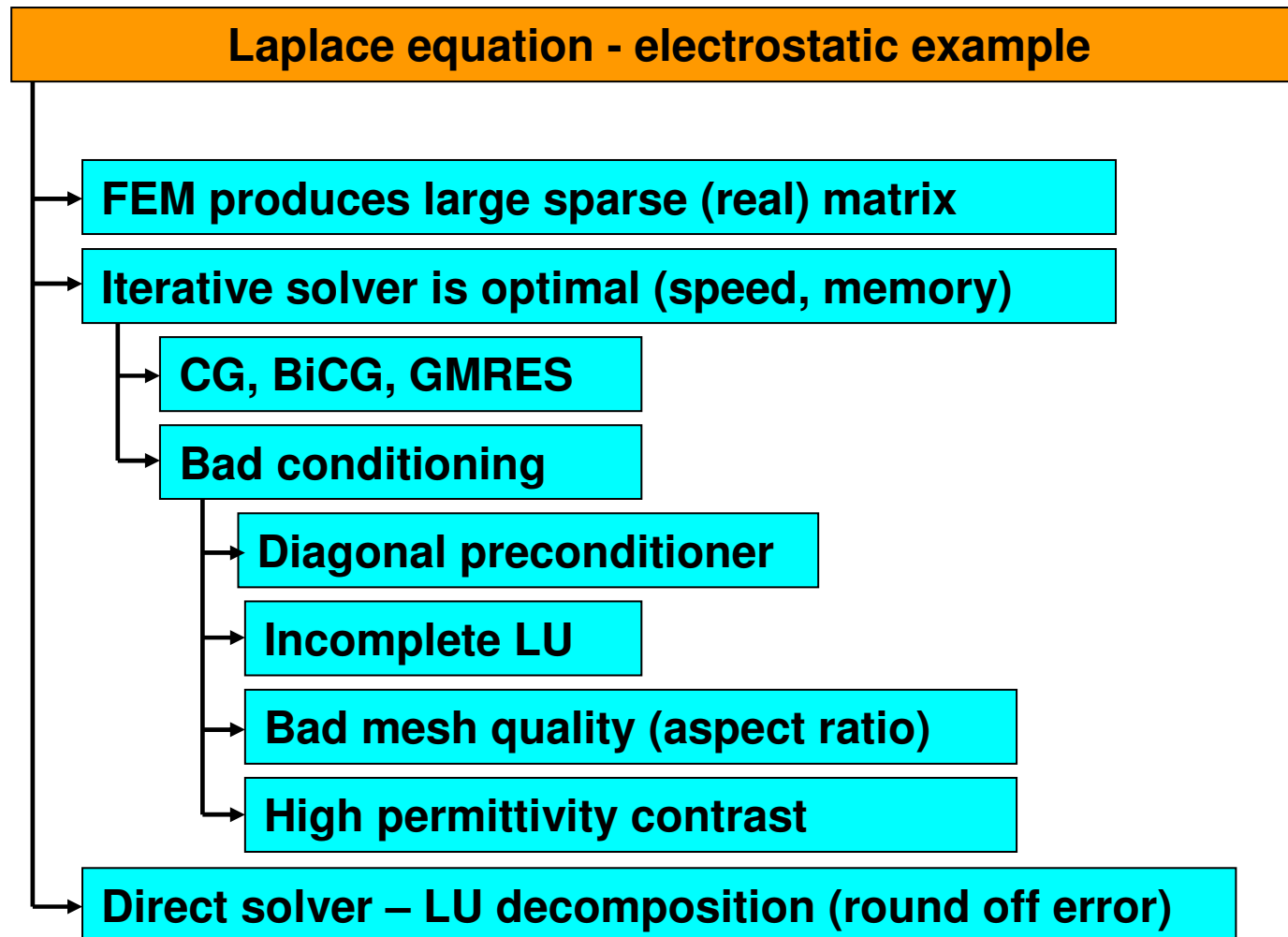
Ne=1399; Ndof=1492

Field plot



Equipotential lines
Electric field vectors

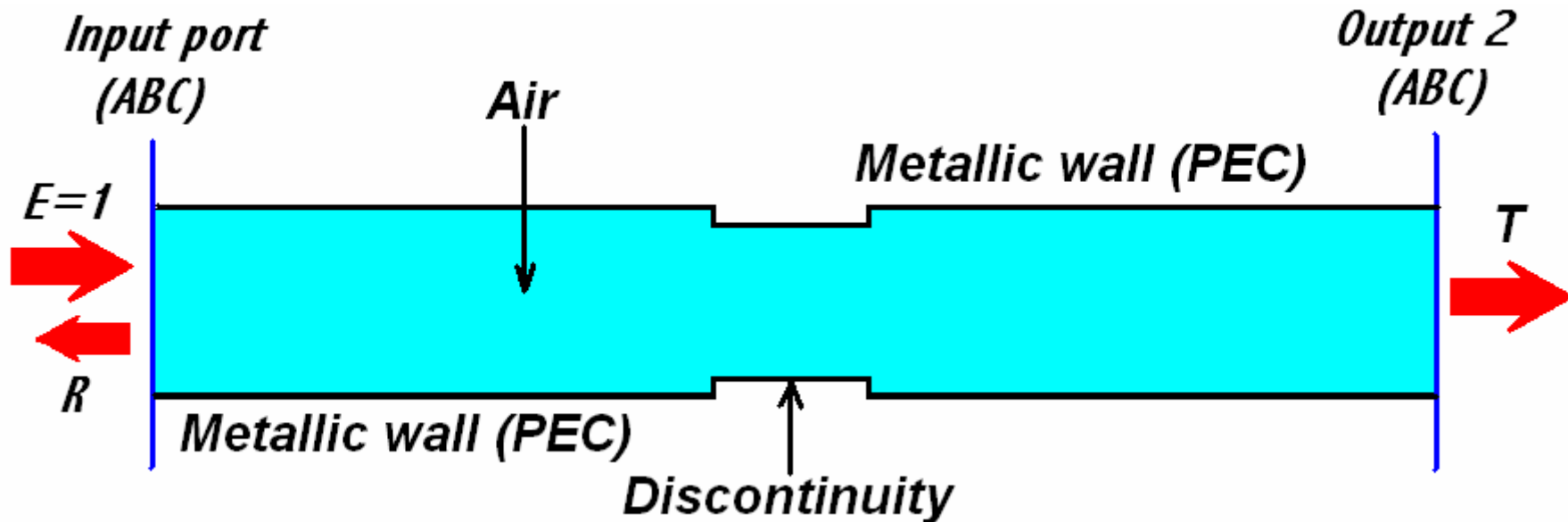
2D electromagnetic analysis



2D electromagnetic analysis

Helmholtz equation – wave propagation example

$$\frac{\partial}{\partial x} \left(\frac{1}{\mu_r} \frac{\partial E_z}{\partial x} \right) - \frac{\partial}{\partial y} \left(\frac{1}{\mu_r} \frac{\partial E_z}{\partial y} \right) + k_0^2 \epsilon_r E_z = 0$$

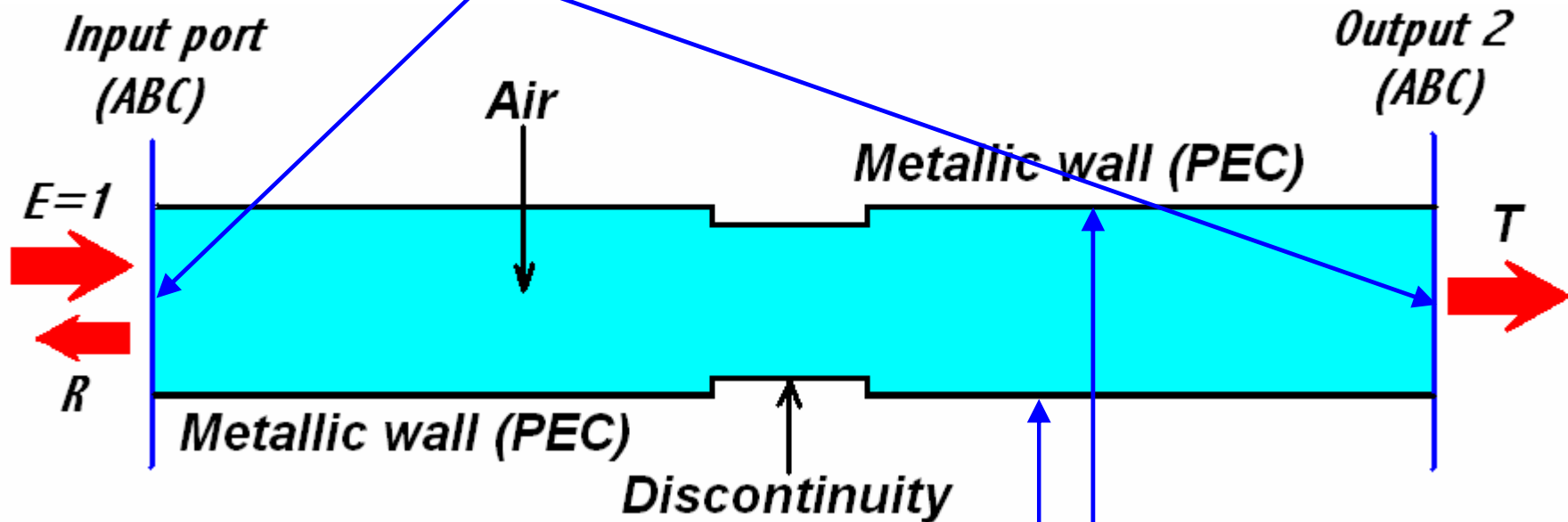


Discontinuity in a parallel-plate waveguide

2D electromagnetic analysis

Helmholtz equation – wave propagation example

$$\vec{e}_z \cdot (\vec{n} \times \sqrt{\mu} \vec{H}) + \sqrt{\epsilon} E_z = 2\sqrt{\epsilon} E_{0z} \leftarrow \text{Low-reflecting boundary (ABC)}$$



Perfect electric conductor (PEC) $\rightarrow \vec{n} \times \vec{E} = 0$

Discontinuity in a parallel-plate waveguide

2D electromagnetic analysis

Helmholtz equation – wave propagation example

$$\vec{e}_z \cdot (\vec{n} \times \sqrt{\mu} \vec{H}) + \sqrt{\epsilon} E_z = 2\sqrt{\epsilon} E_{0z}$$

Low-reflecting boundary (ABC)

$$-\frac{\partial E_z}{\partial n} + j\omega\sqrt{\mu\epsilon} E_z = 2j\omega\sqrt{\mu\epsilon} E_{0z}$$

Neumann BC

$$\alpha \frac{\partial \Phi}{\partial n} + \gamma \Phi = q$$

$$\Phi(x, y) = p(x, y), \quad (x, y) \in \partial\Omega_D$$

Dirichlet BC

$$E_z = 0$$

Perfect electric conductor (PEC)

$$\vec{n} \times \vec{E} = 0$$

Discontinuity in a parallel-plate waveguide

2D electromagnetic analysis

Helmholtz equation – wave propagation example

$$-\frac{\partial E_z}{\partial n} + j\omega\sqrt{\mu\epsilon}E_z = 2j\omega\sqrt{\mu\epsilon}E_{0z} \quad \leftarrow \text{Neumann BC} \quad \rightarrow \quad \alpha \frac{\partial \Phi}{\partial n} + \gamma \Phi = q$$

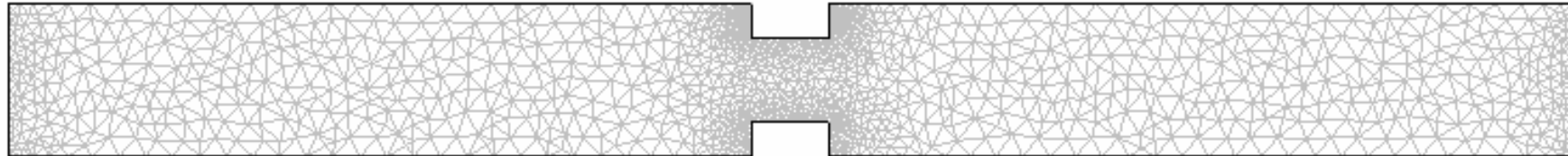
$$F^e(E_z^e) = \frac{1}{2} \iint_{(\Omega^e)} \left[\left(\frac{\partial E_z^e}{\partial x} \right)^2 + \left(\frac{\partial E_z^e}{\partial y} \right)^2 + k^2 E_z^e{}^2 \right] d\Omega + \int_{(\partial\Omega_N^e)} \left(-\frac{jk}{2} E_z^e{}^2 + 2jkE_{0z}^e \right) d\Gamma$$

$$\Phi(x, y) = p(x, y), \quad (x, y) \in \partial\Omega_D \quad \leftarrow \text{Dirichlet BC} \quad \rightarrow \quad E_z = 0$$

Discontinuity in a parallel-plate waveguide

2D electromagnetic analysis

Helmholtz equation – wave propagation example



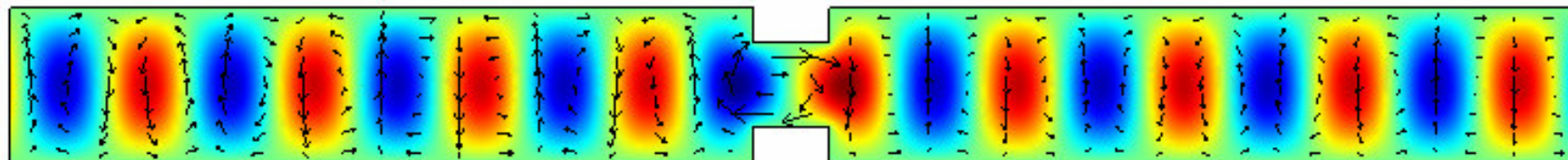
Triangular mesh – Ne=2876; Ndof=1557

$$S_{11} = \frac{\int_{(Port_1)} (E_{zC} - E_{z1}) \cdot E_{z1} \cdot dA_1}{\int_{(Port_1)} E_{z1} \cdot E_{z1} \cdot dA_1}, \quad R = |S_{11}|^2$$

$$S_{12} = \frac{\int_{(Port_2)} E_{zC} \cdot E_{z2} \cdot dA_2}{\int_{(Port_2)} E_{z2} \cdot E_{z2} \cdot dA_2}, \quad T = |S_{12}|^2$$

F=0.78e9 (Hz), Fund. “even” mode

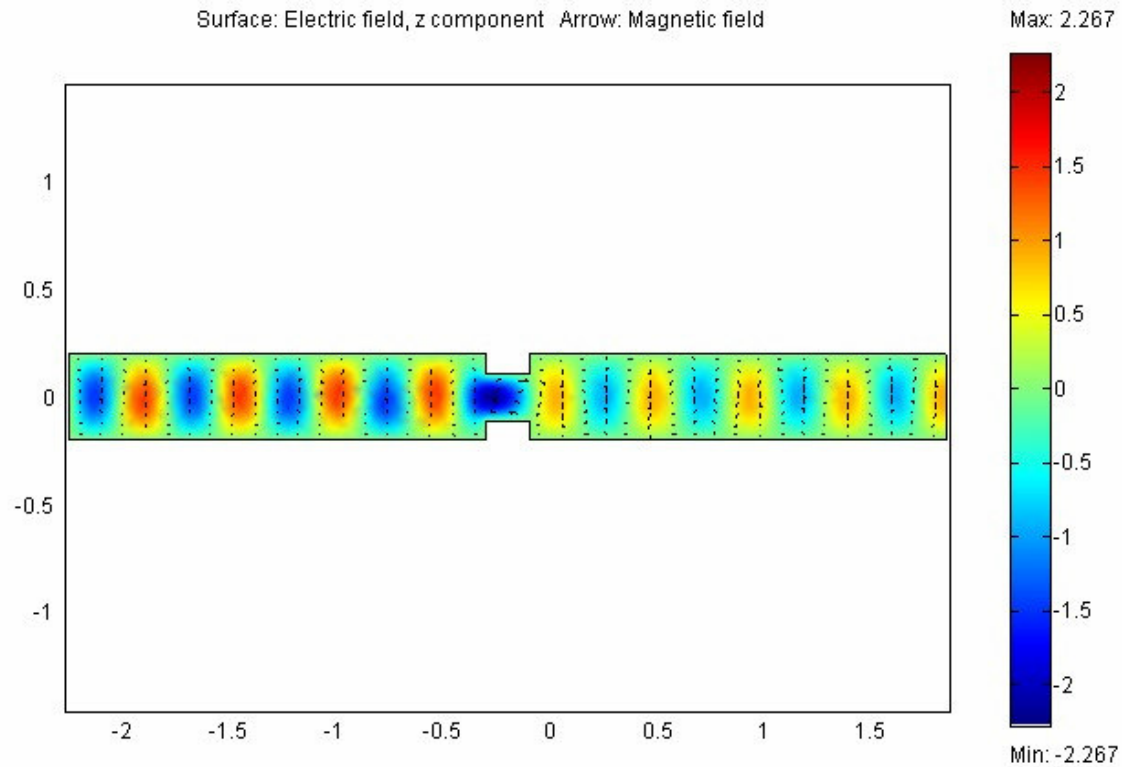
R=21.33%, T=78.67%, R+T=100%



Field results; Ez – color fill ; H vector - arrows

2D electromagnetic analysis

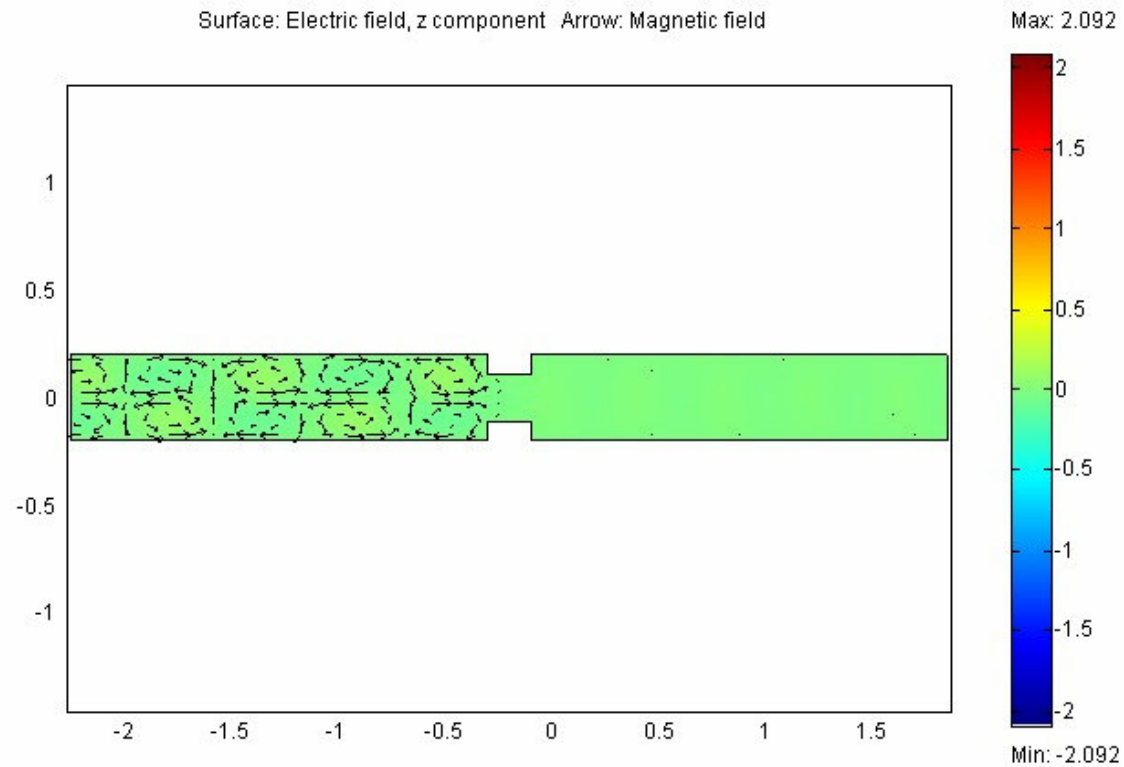
Helmholtz equation – wave propagation example



$F=0.78e9$ (Hz), Fundamental “even” mode: $\cos(y \cdot \pi / 2d)$

2D electromagnetic analysis

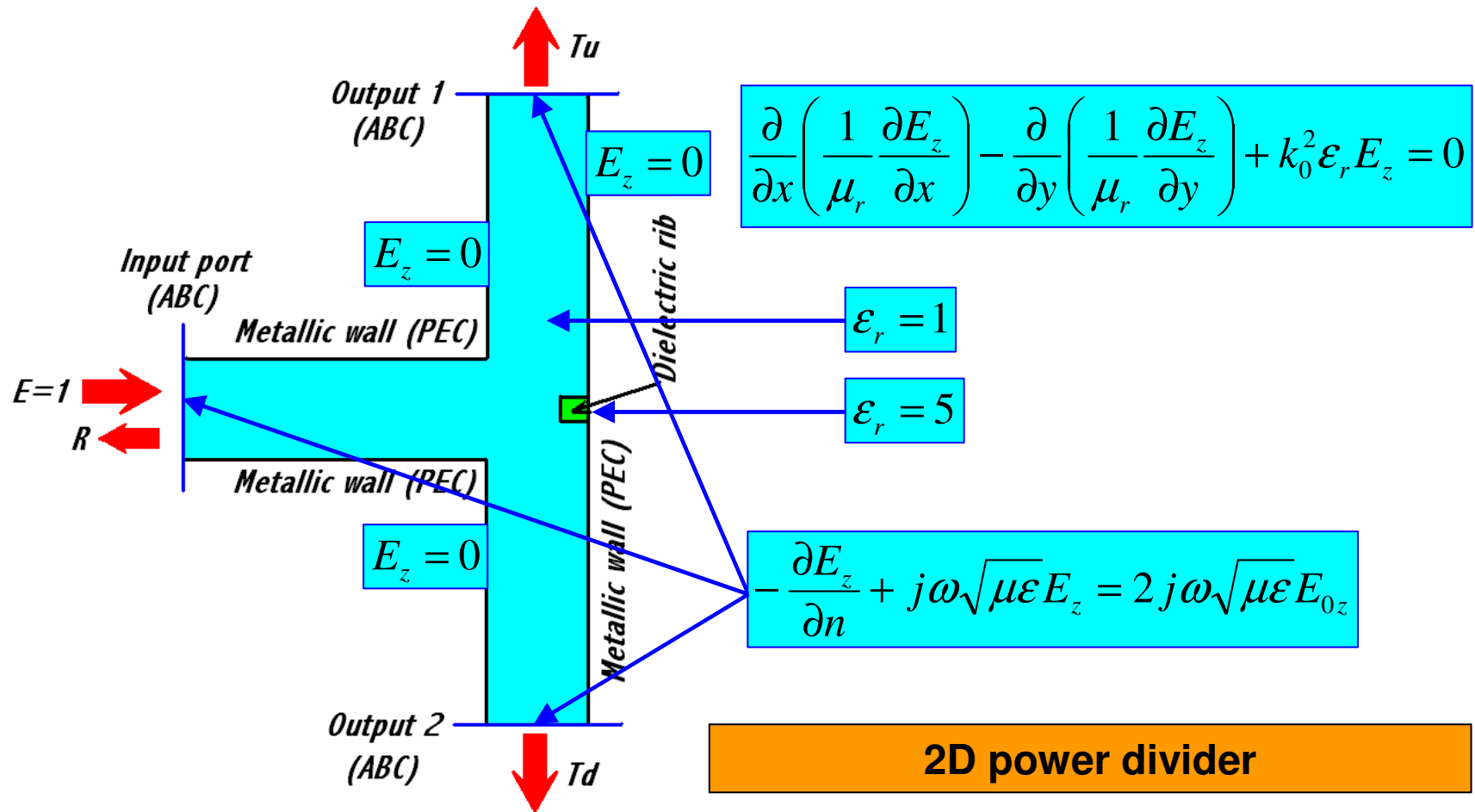
Helmholtz equation – wave propagation example



$F=0.78e9$ (Hz), Fundamental “odd” mode: $\sin(y \cdot \pi/d)$

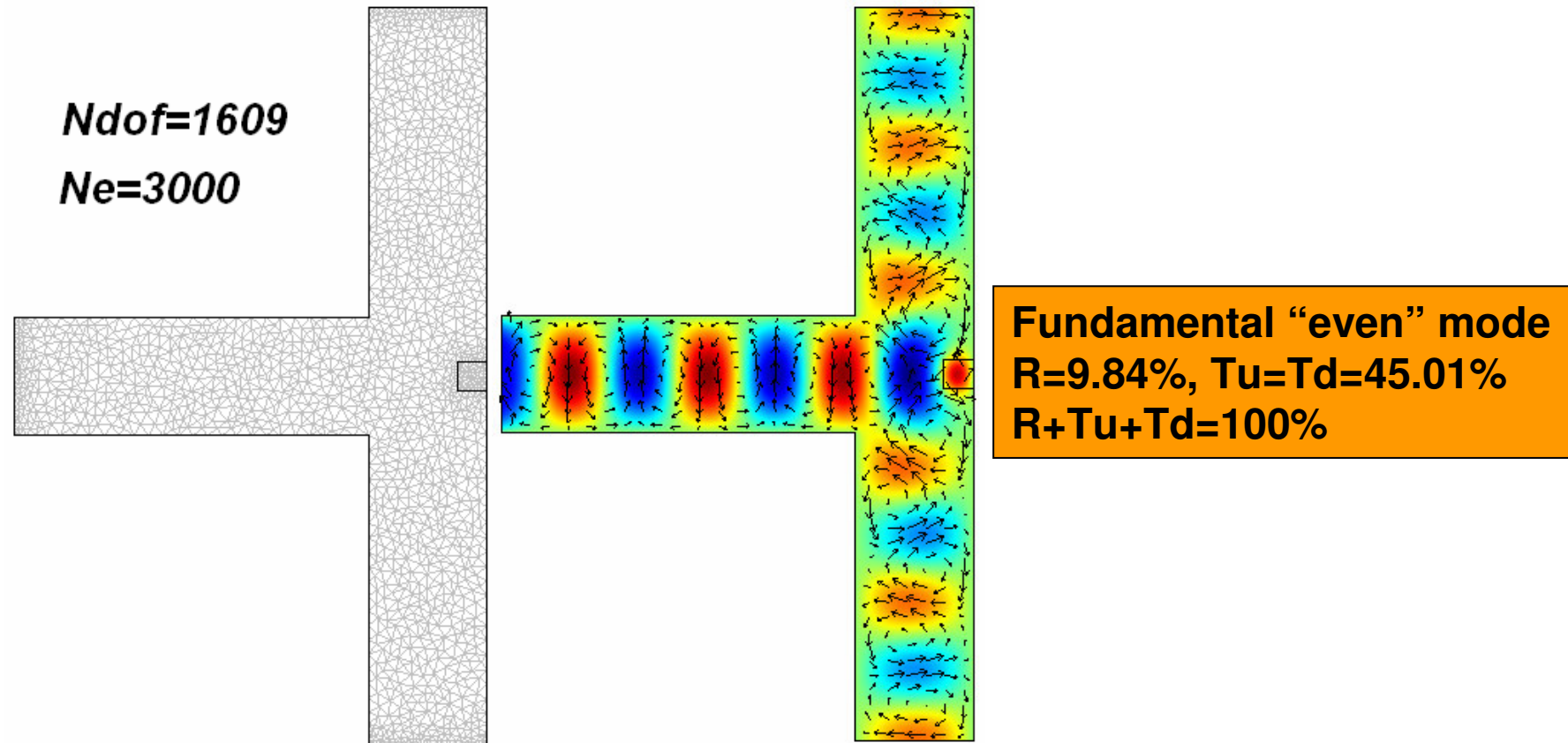
2D electromagnetic analysis

Helmholtz equation – wave propagation example



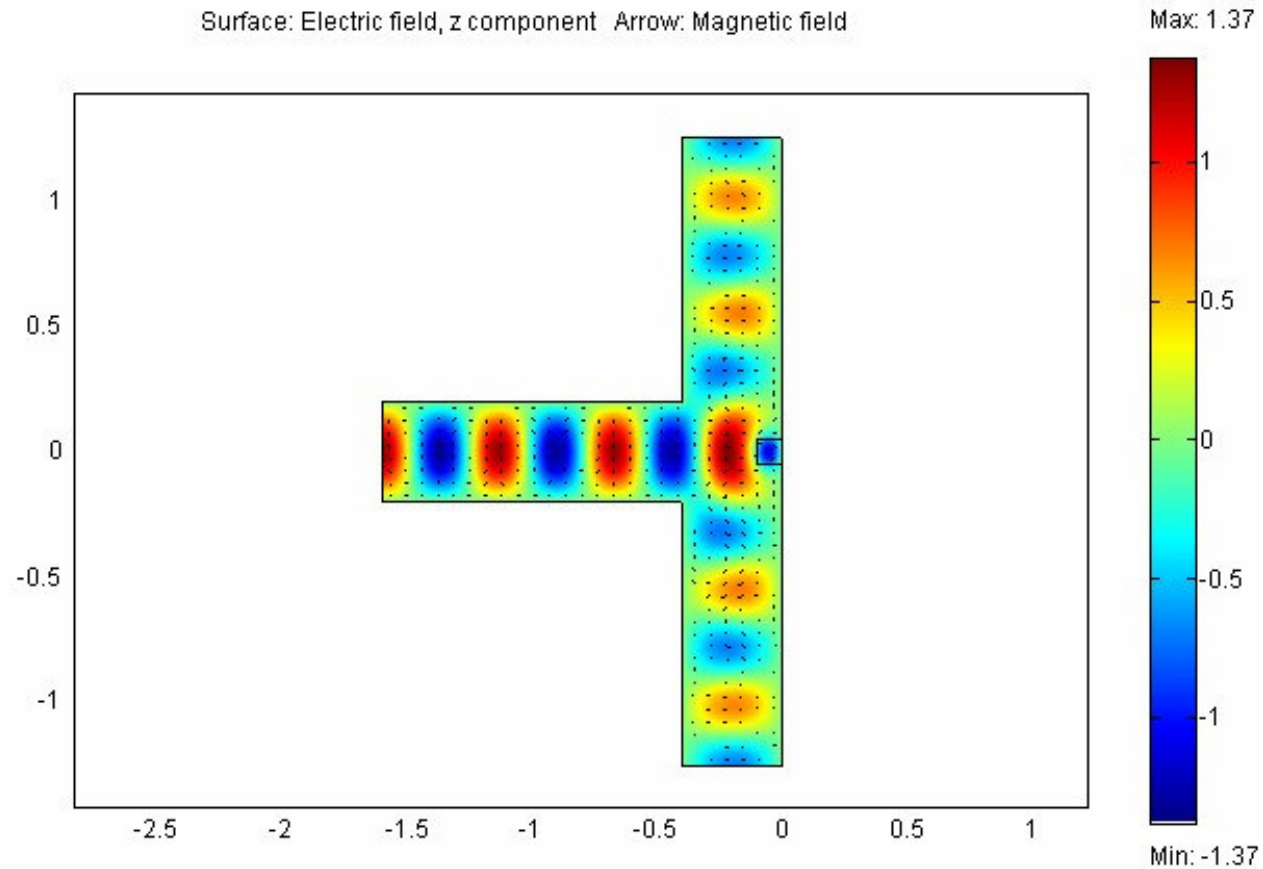
2D electromagnetic analysis

Helmholtz equation – wave propagation example



2D electromagnetic analysis

Helmholtz equation – wave propagation example



2D electromagnetic analysis

Eigenvalue problem – homogeneous waveguide

Propagation in z-direction

$$\vec{E}(x, y, z) = \vec{E}(x, y) \cdot e^{j(\omega t - k_z z)}; \quad \vec{H}(x, y, z) = \vec{H}(x, y) \cdot e^{j(\omega t - k_z z)}$$

TM - modes

$$H_z = 0$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + k_t^2 E_z = 0$$

$$E_z = E_z(x, y): \quad \begin{cases} -jk_z E_y - j\omega\mu H_x = \frac{\partial E_z}{\partial y} \\ -jk_z E_x - j\omega\mu H_y = \frac{\partial E_z}{\partial x} \end{cases}$$

$$k_t = \sqrt{\omega^2 \mu \epsilon - k_z^2}$$

TE - modes

$$E_z = 0$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + k_t^2 H_z = 0$$

$$H_z = H_z(x, y): \quad \begin{cases} -jk_z H_y - j\omega\epsilon E_y = \frac{\partial H_z}{\partial y} \\ -jk_z H_x - j\omega\epsilon E_x = \frac{\partial H_z}{\partial x} \end{cases}$$

2D electromagnetic analysis

Eigenvalue problem – homogeneous waveguide

TM - modes

$$F^e(E_z^e) = \frac{1}{2} \iint_{(\Omega^e)} \left[\left(\frac{\partial E_z^e}{\partial x} \right)^2 + \left(\frac{\partial E_z^e}{\partial y} \right)^2 - k_t^2 E_z^e{}^2 \right] d\Omega$$

$$[K]\{E_z\} = 0$$

$$K_{ij}^e = \iint_{(\Omega^e)} \left(\frac{\partial N_i^e}{\partial x} \frac{\partial N_j^e}{\partial x} + \frac{\partial N_i^e}{\partial y} \frac{\partial N_j^e}{\partial y} - k_t^2 N_i^e N_j^e \right) d\Omega$$

$$K_{ij}^e = A_{ij}^e - k_t^2 B_{ij}^e$$

$$A_{ij}^e = \iint_{(\Omega^e)} \left(\frac{\partial N_i^e}{\partial x} \frac{\partial N_j^e}{\partial x} + \frac{\partial N_i^e}{\partial y} \frac{\partial N_j^e}{\partial y} \right) d\Omega$$

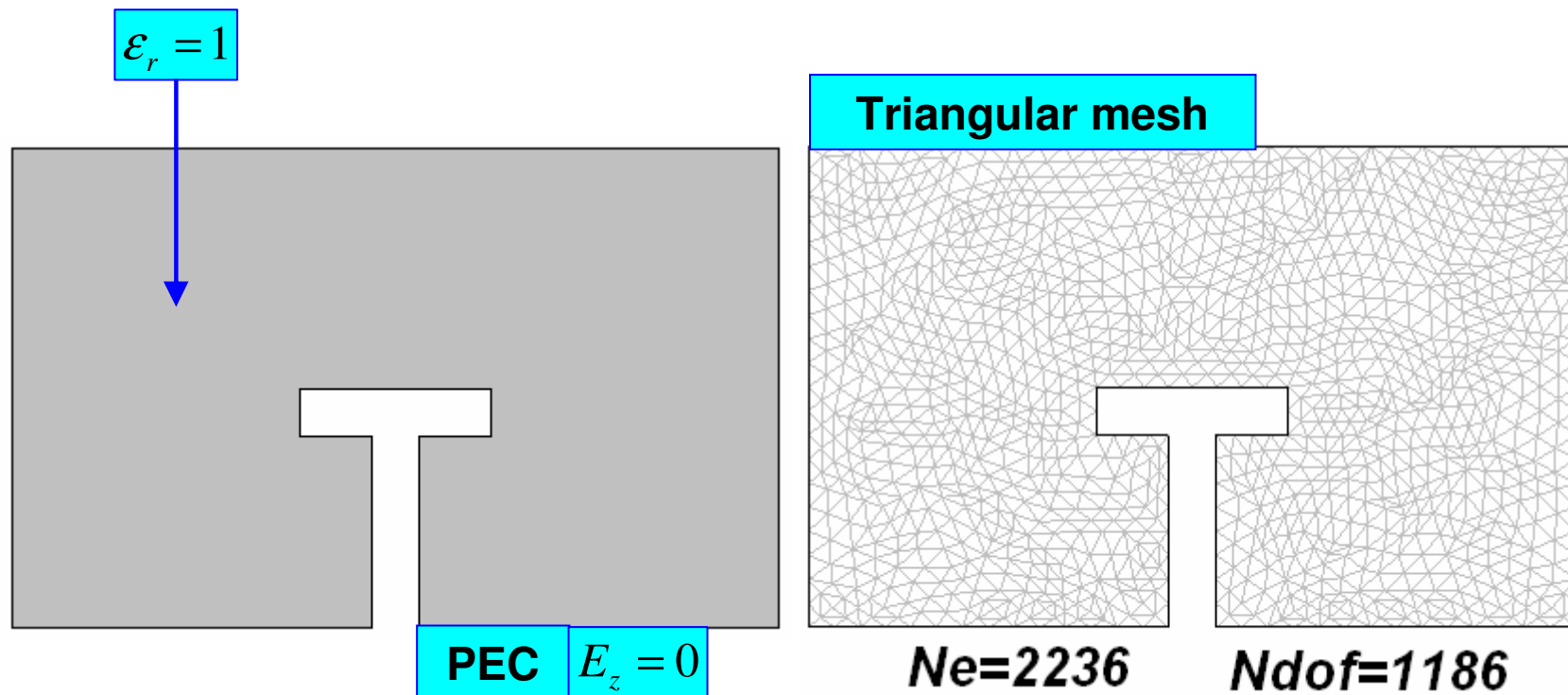
$$B_{ij}^e = \iint_{(\Omega^e)} N_i^e N_j^e d\Omega$$

$$[A]\{E_z\} = k_t^2 [B]\{E_z\}$$

Generalized eigenvalue problem

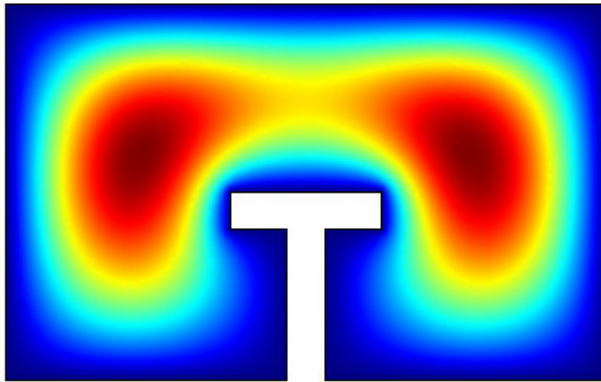
2D electromagnetic analysis

Eigenvalue problem – homogeneous waveguide

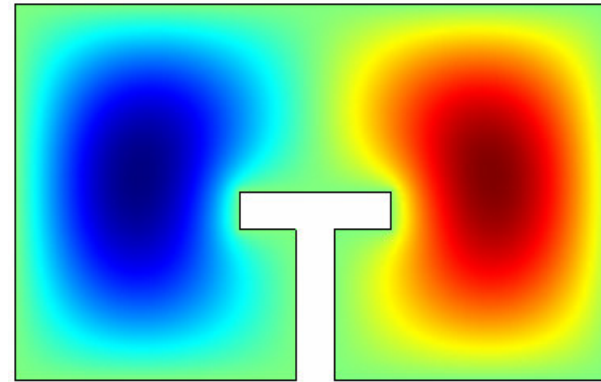


2D electromagnetic analysis

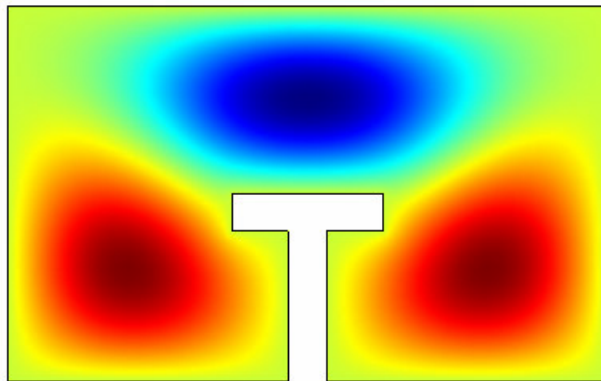
Eigenvalue problem – homogeneous waveguide



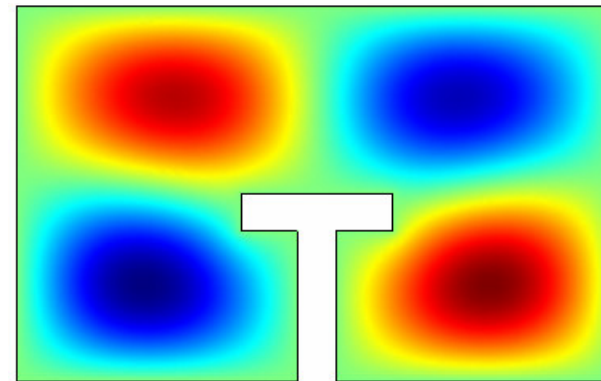
$f=2.61e8$ (cut-off)



$f=2.69e8$ (cut-off)



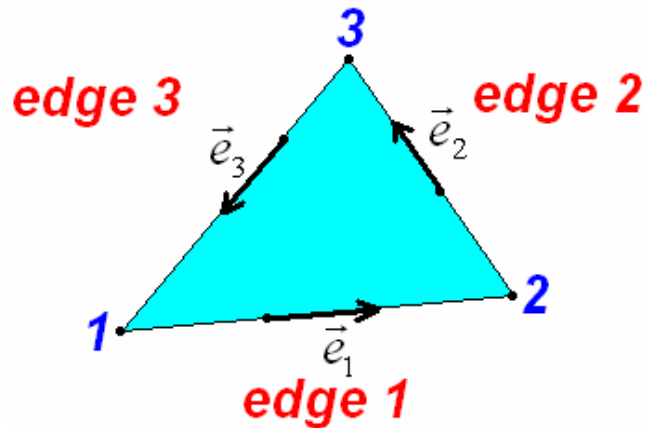
$f=3.34e8$ (cut-off)



$f=3.60e8$ (cut-off)

TM modes: E_z – eigenfield pattern

Vector triangular element



Linear triangular edge element

$$\vec{W}_{12}^e(x, y) = N_1^e \nabla N_2^e - N_2^e \nabla N_1^e$$

$$\nabla \cdot \vec{W}_{12}^e(x, y) = \nabla \cdot (N_1^e \nabla N_2^e) - \nabla \cdot (N_2^e \nabla N_1^e) = 0$$

$$\begin{aligned} \nabla \times \vec{W}_{12}^e(x, y) &= \nabla \times (N_1^e \nabla N_2^e) - \nabla \times (N_2^e \nabla N_1^e) = \\ &= 2 \nabla N_1^e \times \nabla N_2^e \end{aligned}$$

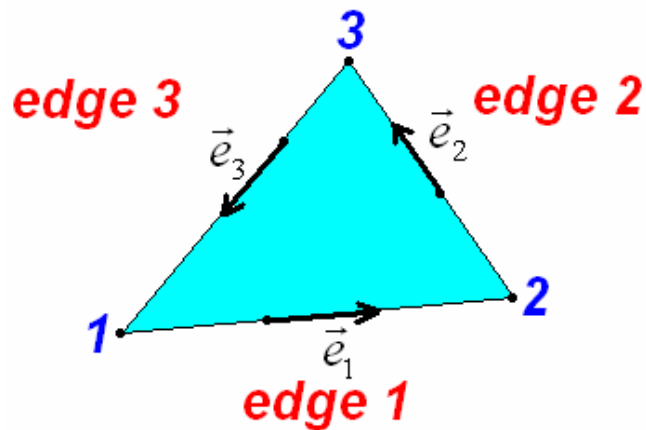
$$\vec{e}_1 \cdot \nabla N_1^e = \frac{\partial N_1^e}{\partial e_1} = -\frac{1}{l_1^e} \quad \vec{e}_1 \cdot \nabla N_2^e = \frac{\partial N_2^e}{\partial e_1} = \frac{1}{l_1^e}$$

$$\vec{e}_1 \cdot \vec{W}_{12}^e = \frac{N_1^e + N_2^e}{l_1^e} = \frac{1}{l_1^e}, \quad \forall (x, y) \in \partial\Omega_1^e - \text{edge}1$$

$$N_1^e(x, y) = 0, \quad \forall (x, y) \in \partial\Omega_2^e \Rightarrow \vec{e}_2 \cdot \vec{W}_{12}^e = 0$$

$$N_2^e(x, y) = 0, \quad \forall (x, y) \in \partial\Omega_3^e \Rightarrow \vec{e}_3 \cdot \vec{W}_{12}^e = 0$$

Vector triangular element



Linear triangular edge element

Dimensionless and normalized vector shape functions:

$$\vec{V}_1^e = \vec{W}_{12}^e \cdot l_1^e = (N_1^e \nabla N_2^e - N_2^e \nabla N_1^e) \cdot l_1^e$$

$$\vec{V}_2^e = \vec{W}_{23}^e \cdot l_2^e = (N_2^e \nabla N_3^e - N_3^e \nabla N_2^e) \cdot l_2^e$$

$$\vec{V}_3^e = \vec{W}_{31}^e \cdot l_3^e = (N_3^e \nabla N_1^e - N_1^e \nabla N_3^e) \cdot l_3^e$$

Vector field approximation over the element:

$$\vec{E}^e(x, y) = \sum_{i=1}^3 \vec{V}_i^e(x, y) \cdot E_i^e, \quad E_i^e = \vec{E}^e \cdot \vec{e}_i$$

2D electromagnetic analysis

Eigenvalue problem – inhomogeneous waveguide

- Hybrid propagation modes (pure TE or TM do not exist)
- Hz-Ez formulation possible but inaccurate (spurious modes)
- Full vector formulation + edge element = accurate solution

$$\nabla \times \left(\frac{1}{\mu_r} \nabla \times \vec{E} \right) - k_0^2 \epsilon_r \vec{E} = 0$$

$$\vec{n} \times \vec{E} = 0 \text{ (PEC)}$$

$$\vec{n} \times (\nabla \times \vec{E}) = 0 \text{ (PMC)}$$

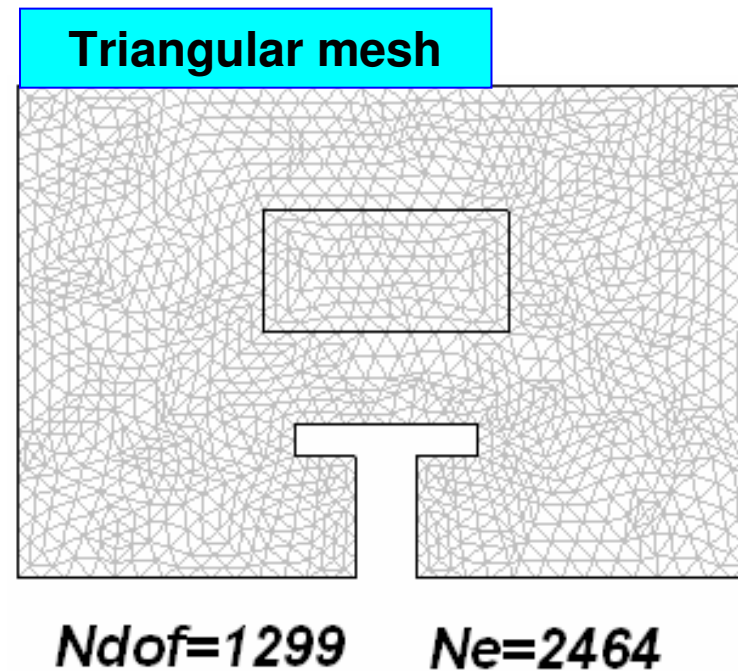
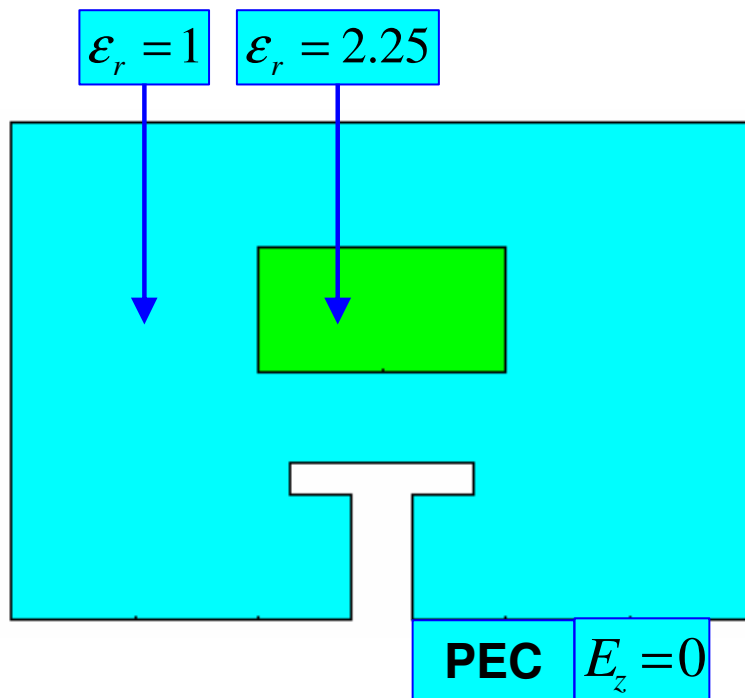
$$F(\vec{E}) = \frac{1}{2} \iint_{(\Omega)} \left[\frac{1}{\mu_r} (\nabla \times \vec{E}) \cdot (\nabla \times \vec{E})^* - k_0^2 \epsilon_r \vec{E} \cdot \vec{E}^* \right] d\Omega$$

$$F(\vec{E}) = \frac{1}{2} \iint_{(\Omega)} \left[\frac{1}{\mu} (\nabla_t \times \vec{E}_t) \cdot (\nabla_t \times \vec{E}_t)^* - k_0^2 \epsilon_r \vec{E} \cdot \vec{E}^* + \frac{1}{\mu_r} (\nabla_t E_z + jk_z \vec{E}_t) \cdot (\nabla_t E_z + jk_z \vec{E}_t)^* \right] d\Omega$$

$$\vec{E}(x, y, z) = \vec{E}(x, y) \cdot e^{-jk_z z}$$

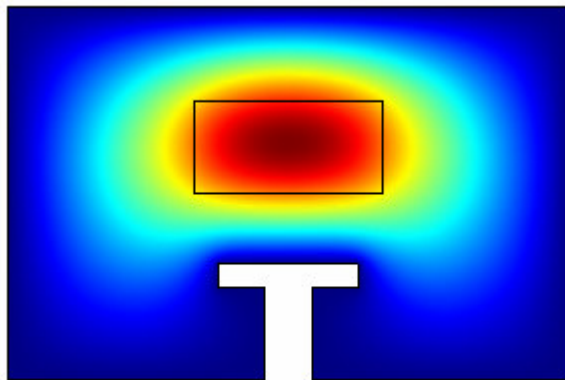
2D electromagnetic analysis

Eigenvalue problem – inhomogeneous waveguide

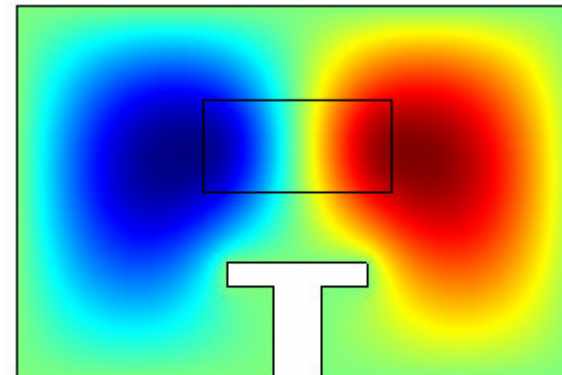


2D electromagnetic analysis

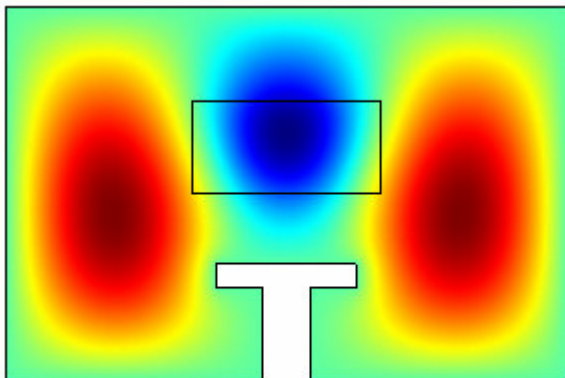
Eigenvalue problem – inhomogeneous waveguide



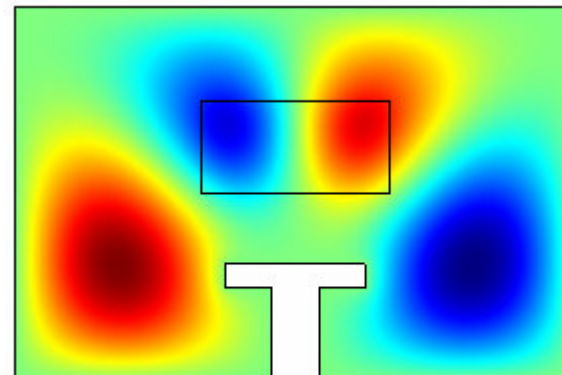
$f=2.31e8$ (cut-off)



$f=3.10e8$ (cut-off)



$f=3.83e8$ (cut-off)



$f=4.43e8$ (cut-off)

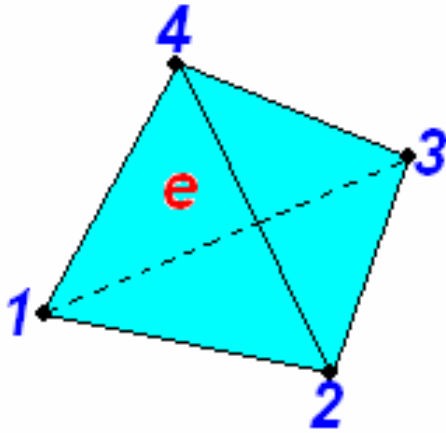
2D electromagnetic analysis

Eigenvalue problem – inhomogeneous waveguide

Vector edge element – basic advantages compared to nodal element:

1. Spurious solutions are removed
2. Accurate treatment of conducting edges and sharp corners
3. Accurate treatment of edges and sharp corners of inhomogeneous dielectric

Nodal tetrahedral element



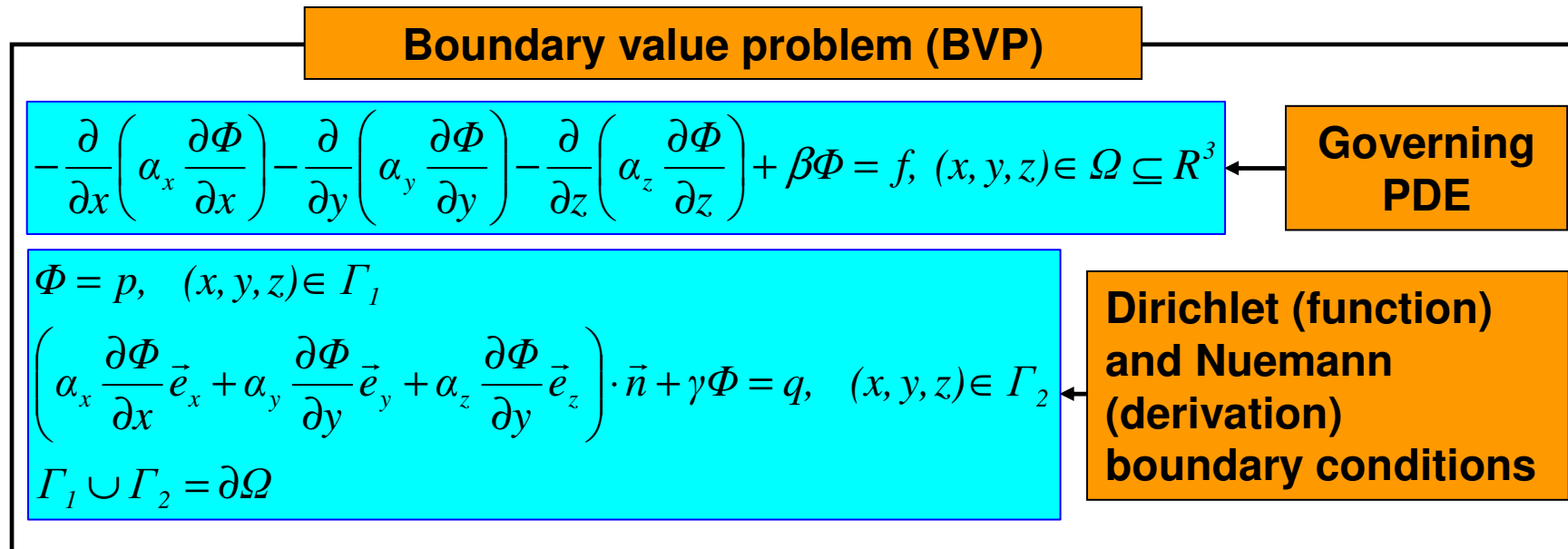
Linear tetrahedral nodal element

$$\Phi^e(x,y) = a^e + b^e x + c^e y + d^e z$$

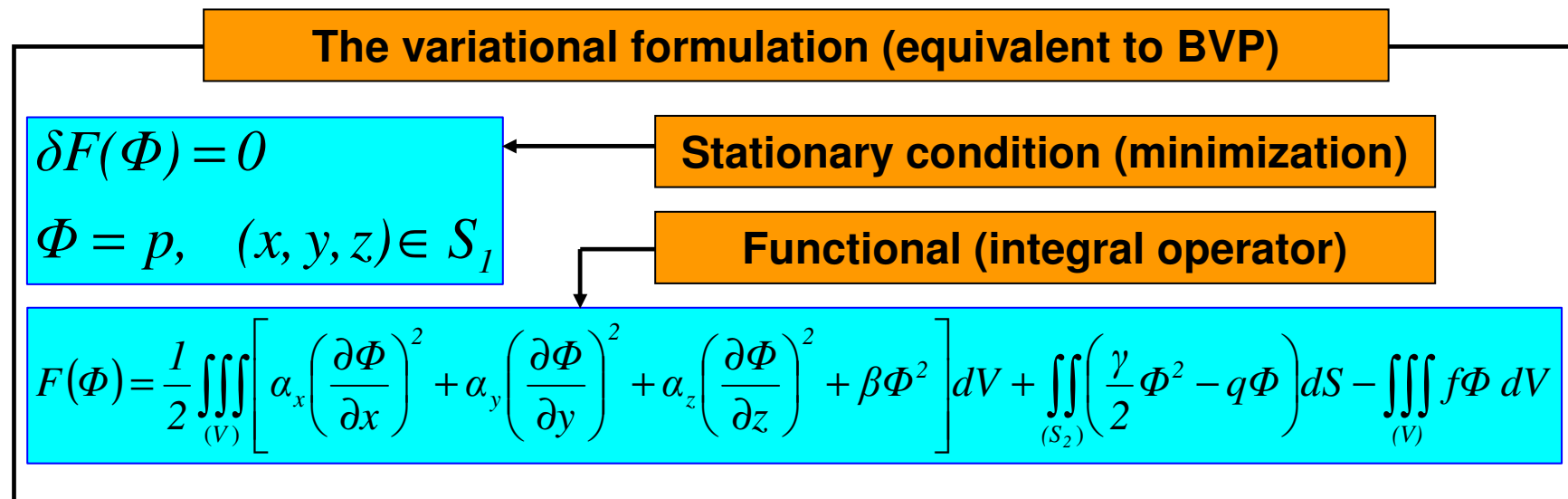
$$\Phi^e(x,y,z) = \sum_{j=1}^4 N_j^e(x,y,z) \cdot \Phi_j^e$$

$$N_j^e(x,y,z) = \frac{1}{6V^e} (a_j^e + x b_j^e + y c_j^e + z d_j^e), \quad j = 1, 2, 3, 4$$

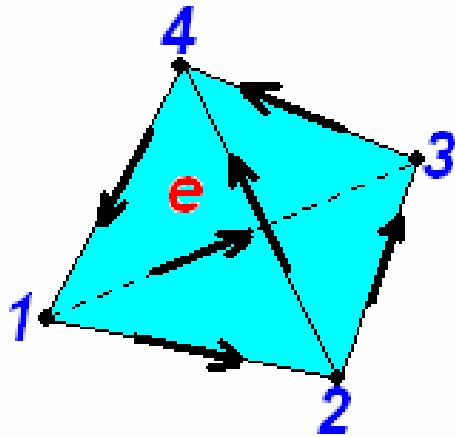
3D electromagnetic analysis



3D electromagnetic analysis



Vector tetrahedral element



Linear tetrahedral edge element

$$\vec{W}_{12}^e(x, y, z) = N_1^e \nabla N_2^e - N_2^e \nabla N_1^e$$

$$\nabla \cdot \vec{W}_{12}^e(x, y) = \nabla \cdot (N_1^e \nabla N_2^e) - \nabla \cdot (N_2^e \nabla N_1^e) = 0$$

$$\begin{aligned} \nabla \times \vec{W}_{12}^e(x, y) &= \nabla \times (N_1^e \nabla N_2^e) - \nabla \times (N_2^e \nabla N_1^e) = \\ &= 2 \nabla N_1^e \times \nabla N_2^e \end{aligned}$$

$$\vec{e}_1 \cdot \nabla N_1^e = \frac{\partial N_1^e}{\partial e_1} = -\frac{1}{l_1^e}$$

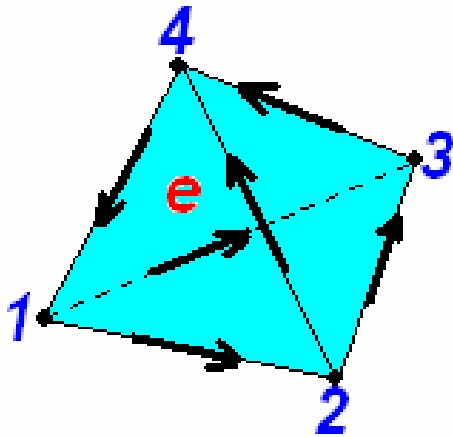
$$\vec{e}_1 \cdot \nabla N_2^e = \frac{\partial N_2^e}{\partial e_1} = \frac{1}{l_1^e}$$

$$\vec{e}_1 \cdot \vec{W}_{12}^e = \frac{N_1^e + N_2^e}{l_1^e} = \frac{1}{l_1^e}, \quad \forall (x, y) \in \partial\Omega_1^e - \text{edge } 1$$

$$N_1^e(x, y) = 0, \quad \forall (x, y) \in \partial\Omega_2^e \Rightarrow \vec{e}_2 \cdot \vec{W}_{12}^e = 0$$

$$N_2^e(x, y) = 0, \quad \forall (x, y) \in \partial\Omega_3^e \Rightarrow \vec{e}_3 \cdot \vec{W}_{12}^e = 0$$

Vector tetrahedral element



Linear tetrahedral edge element

Dimensionless and normalized vector shape functions:

$$\vec{V}_1^e = \vec{W}_{12}^e \cdot l_1^e = (N_1^e \nabla N_2^e - N_2^e \nabla N_1^e) \cdot l_1^e$$

$$\vec{V}_2^e = \vec{W}_{23}^e \cdot l_2^e = (N_2^e \nabla N_3^e - N_3^e \nabla N_2^e) \cdot l_2^e$$

$$\vec{V}_3^e = \vec{W}_{31}^e \cdot l_3^e = (N_3^e \nabla N_1^e - N_1^e \nabla N_3^e) \cdot l_3^e$$

$$\vec{V}_4^e = \vec{W}_{34}^e \cdot l_4^e = (N_3^e \nabla N_4^e - N_4^e \nabla N_3^e) \cdot l_4^e$$

$$\vec{V}_5^e = \vec{W}_{24}^e \cdot l_5^e = (N_2^e \nabla N_4^e - N_4^e \nabla N_2^e) \cdot l_5^e$$

$$\vec{V}_6^e = \vec{W}_{41}^e \cdot l_6^e = (N_4^e \nabla N_1^e - N_1^e \nabla N_4^e) \cdot l_6^e$$

Vector field approximation over the element:

$$\vec{E}^e(x, y, z) = \sum_{i=1}^6 \vec{V}_i^e(x, y, z) \cdot E_i^e, \quad E_i^e = \vec{E}^e \cdot \vec{e}_i$$

3D electromagnetic analysis

Wave equation - 3D analysis of photonic crystal waveguide

Governing PDE

$$\nabla \times \left(\frac{1}{\mu_r} \nabla \times \vec{E} \right) - k_0^2 \epsilon_r \vec{E} = 0$$

Boundary conditions

$$\vec{n} \times \vec{E} = 0 \text{ (PEC)} \quad ; \quad \vec{n} \times (\nabla \times \vec{E}) = 0 \text{ (PMC)}$$

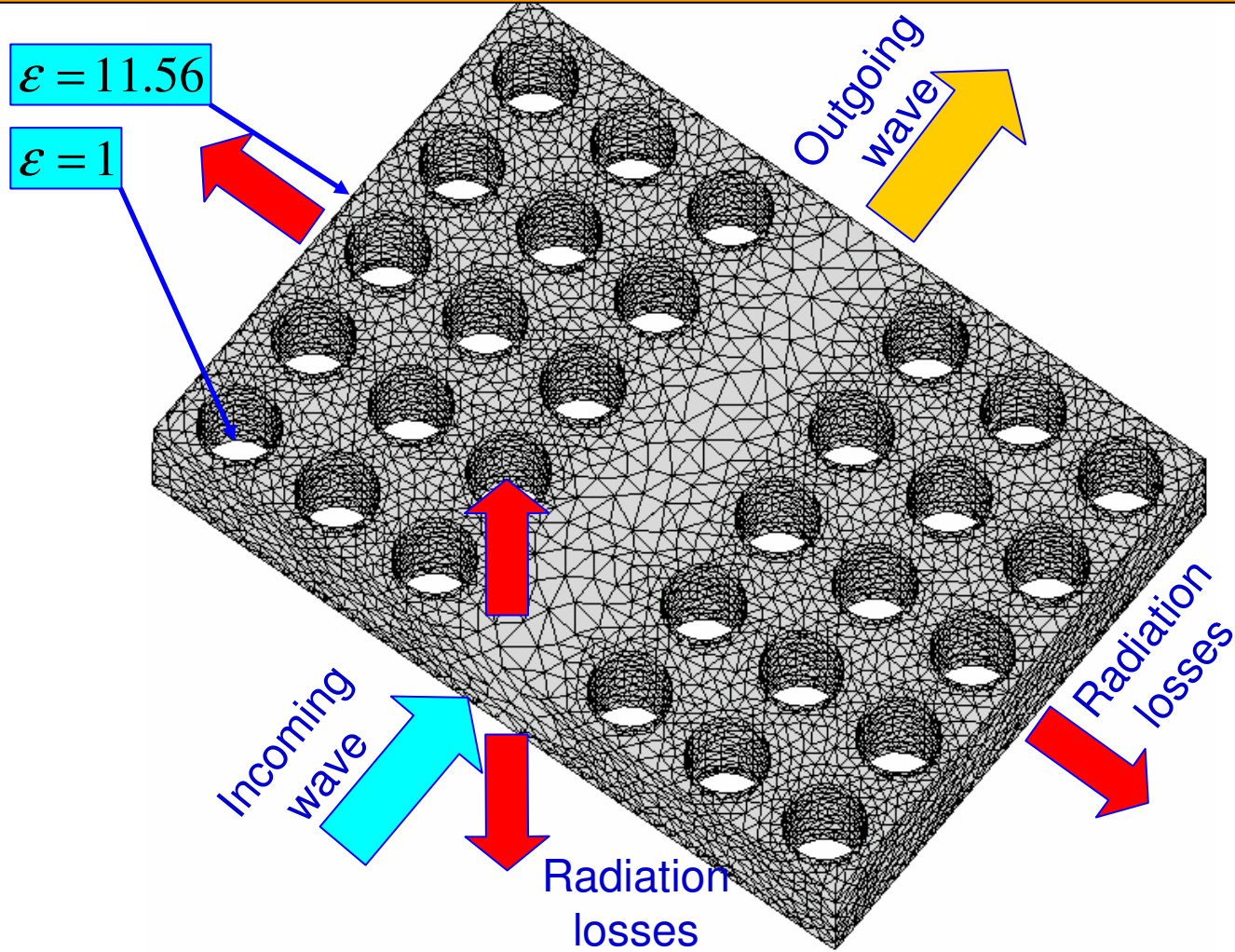
$$\frac{1}{\mu_r} \vec{n} \times (\nabla \times \vec{E}) + \gamma_e \vec{n} \times (\vec{n} \times \vec{E}) = \vec{U}$$

Functional

$$F(\vec{E}) = \frac{1}{2} \iiint_{(V)} \left[\frac{1}{\mu_r} (\nabla \times \vec{E}) \cdot (\nabla \times \vec{E}) - k_0^2 \epsilon_r \vec{E} \cdot \vec{E} \right] dV + \iint_{(S_2)} \left[\frac{\gamma_e}{2} (\vec{n} \times \vec{E}) \cdot (\vec{n} \times \vec{E}) + \vec{E} \cdot \vec{U} \right] dS$$

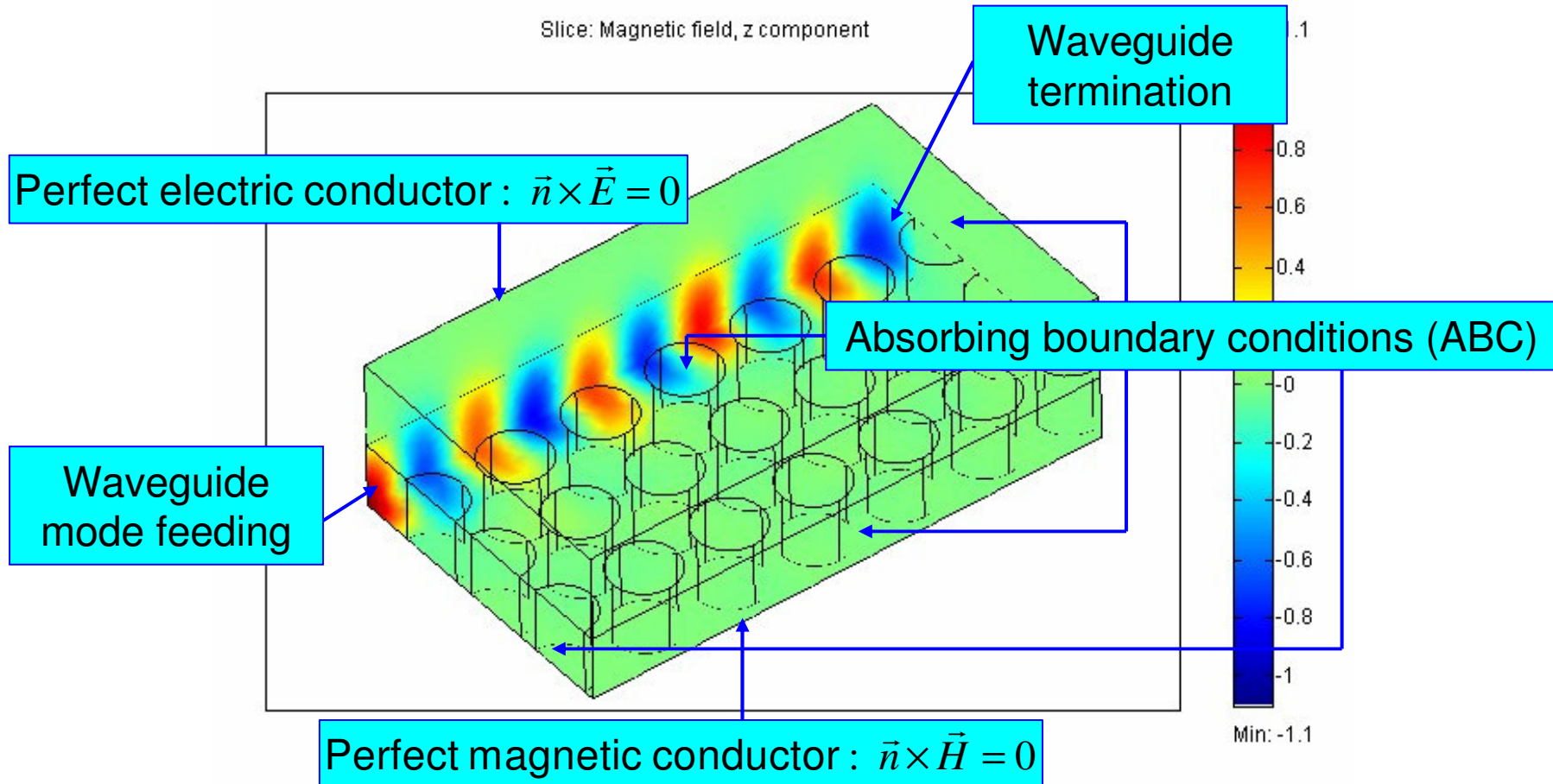
3D electromagnetic analysis

Wave equation - 3D analysis of photonic crystal waveguide



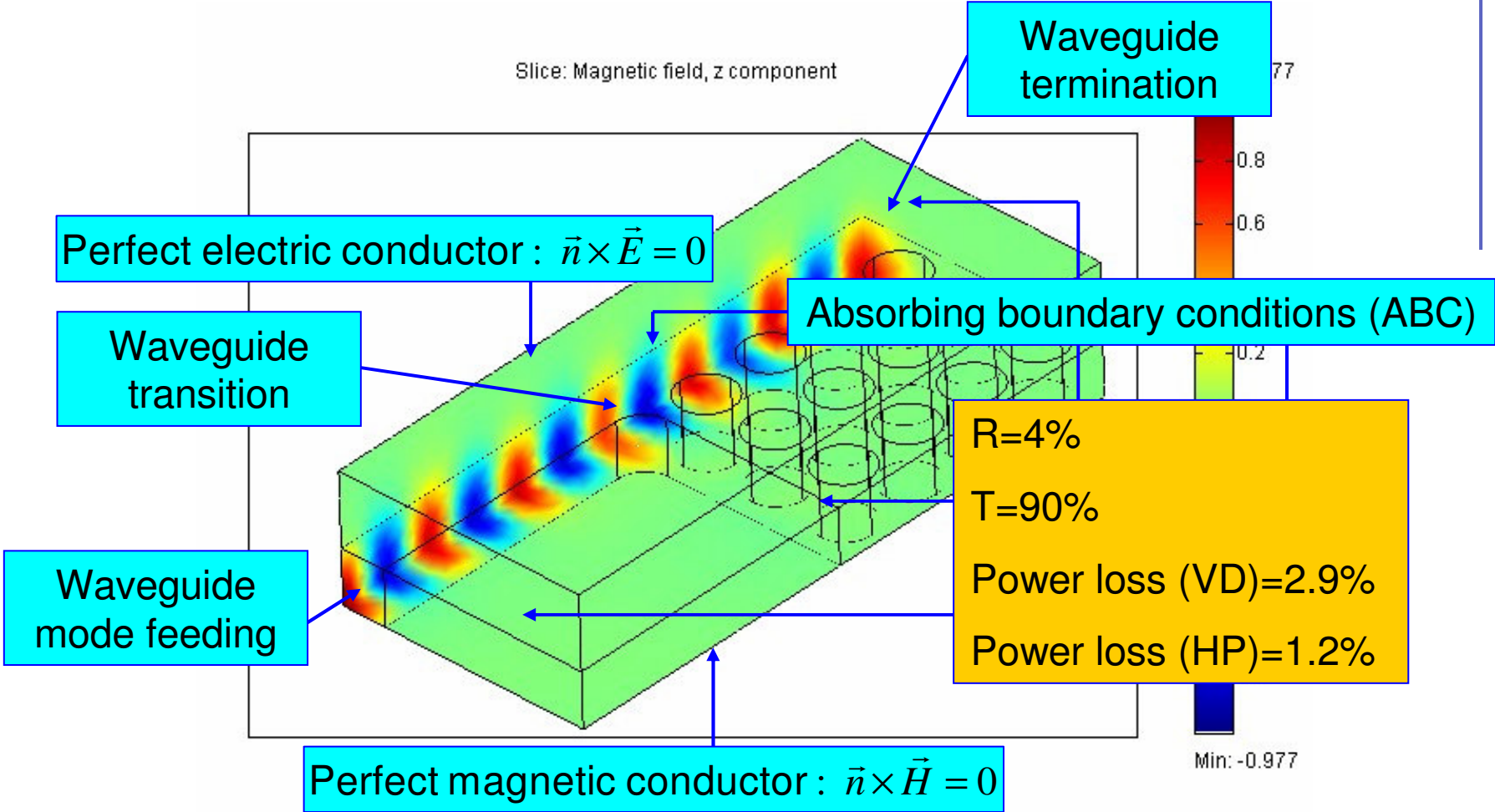
3D electromagnetic analysis

Wave equation - 3D analysis of photonic crystal waveguide



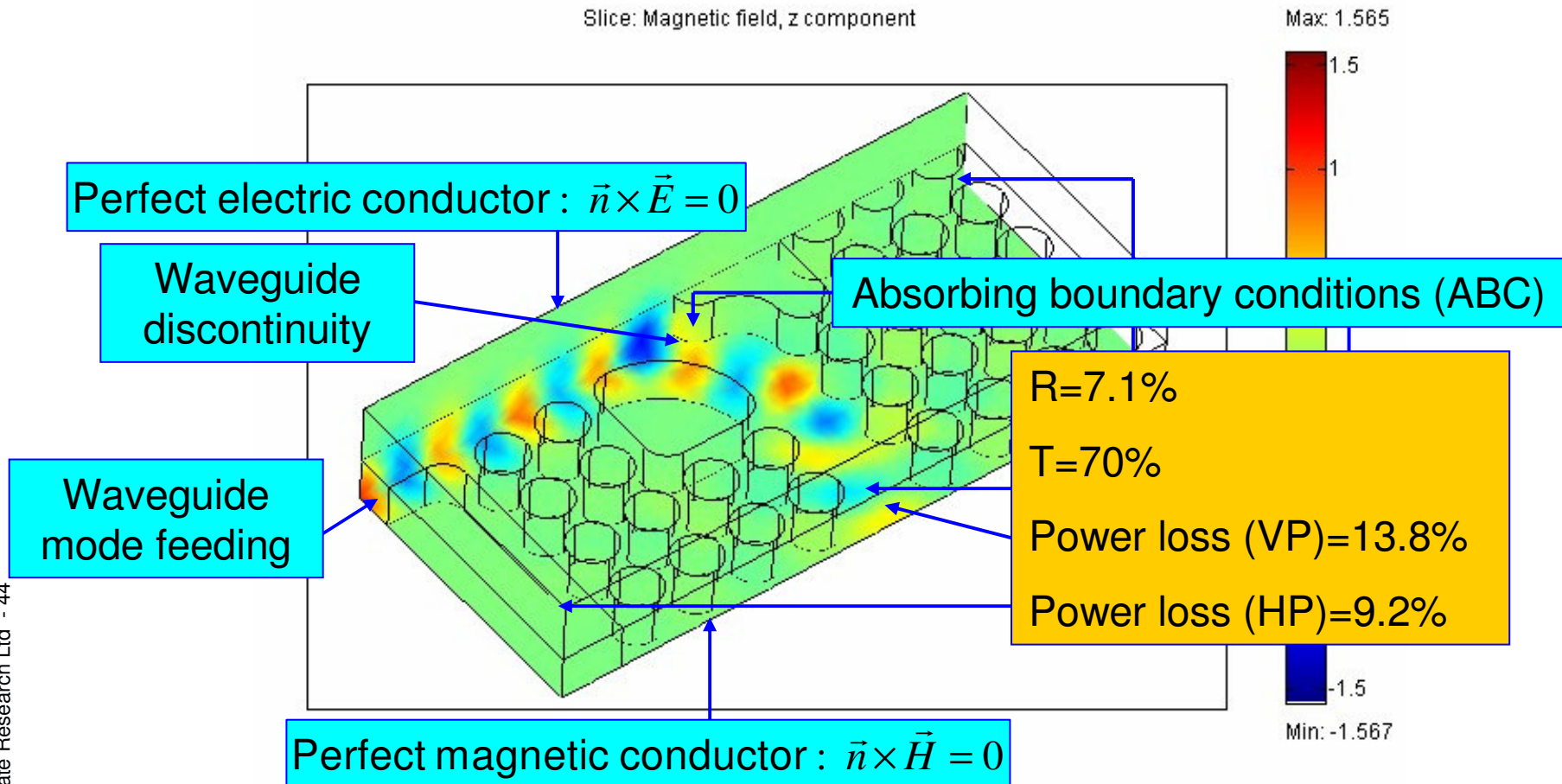
3D electromagnetic analysis

Wave equation - 3D analysis of photonic crystal coupler



3D electromagnetic analysis

Wave equation - 3D analysis of photonic crystal power divider



Conclusions

- We have illustrated FEM application to various electromagnetic problems
- Several different formulations of elements (nodal and edge) has been analyzed and used in appropriate examples
- Obviously FEM is very powerful simulation tool for electromagnetic analysis
- Since FEM application to electromagnetic simulations has considerable tradition, there are very many commercial software packages based on FEM (ANSYS, ANSOFT, INFOLYTICA, FEMLAB etc.)

ABB

