Exercise 19 - OLD EXAM, FDTD

A 1D wave propagation may be considered by the coupled differential equations

$$\frac{\partial u}{\partial x} + a\frac{\partial v}{\partial t} = 0 \quad (1)$$

$$\frac{\partial v}{\partial x} + b\frac{\partial u}{\partial t} = 0 \quad (2)$$

a) 2 points: Derive the decoupled differential equation

$$c^2\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 v}{\partial t^2} = 0 \quad (3)$$

and give c in terms of a,b.

b) 3 points: One possible solution of (3) has the form

$$v\left(x,t\right) = \Re e\left\{V_0 e^{i(kx-\omega t)}\right\} \quad (4)$$

where V_0 denotes the complex amplitude, k the wavenumber and ω the angular frequency of the wave. Demonstrate that the equations (1) - (3) are fulfilled by (4) and compute k for a given ω.

c) 2 points: Compute $u(x,t)$, when the solution (4) holds.

d) 3 points: Derive a FDTD scheme that discretises (3) using central differences and give the stability criteria for a discretisation Δx, Δt in x and t.

Total: 10 points

Exercise 20 - OLD EXAM, Electrostatics + FD

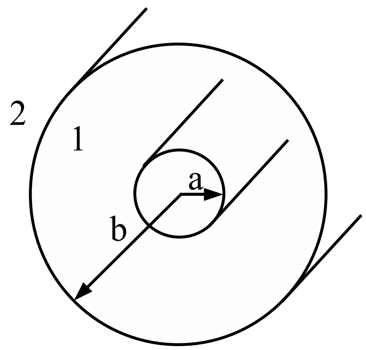


Figure 1: A linear triangular element is presented.

A coaxial cable is given in the figure above. The total charge per length on the inner conductor with radius a is $+\sigma$ and the total charge per length on the outer conductor with radius b is $-\sigma$. The gap between the conductors is filled with a dielectric material with the permittivity value of $\varepsilon = \varepsilon_r \varepsilon_0$.

a) 2 points: State the Gauss' Law in Electrostatics and describe its physical meaning.

b) 2 points: Determine the electrical field in regions 1 and 2.

c) 1 points: By using the result in part b, find the potential difference between the two conductors.

a) 2 points: Derive the governing equation for the radial dependence of the potential distribution between the conductors and find an update scheme by using central difference formula. Set the first boundary value as $V(r = a) = V_0$ and use the result from c) to compute the second boundary value $V(r = b)$.

Total: 10 points

Exercise 21 - OLD EXAM, FEM Solutions

A horn antenna with an aperture of 1 m is feed by an air filled waveguide of l.5 m length, 0.4 m height and 0.1 m width. The TE_{02} mode is simulated with COMSOL Multiphysics as depicted in Fig. 6.1(a)-(f). The figures show the E_z fields with a constant color scale.

a) 2 points: Compute the cutoff frequencies for the first two modes.

b) 3 points: Estimate the frequency of the depicted TE_{02} mode from Fig. 6.1(a)-(f)

c) 1 point: Which of the six depicted results in figures 6.1(a)-(f) models the described problem most accurately?

d) 5 points: The other five models contain one mistake each - either by definition of boundary condition or material properties. Name the mistakes!

e) 2 points: Sketch a suitable mesh for the problem and explain it.

f) 1 point: How can the computational effort be reduced without changing the mesh density? Give details.

Total: 14 points

Exercise 22 - OLD EXAM, Multiple Choice Part

Answer the following questions either in short notes or by circling the right statement.

a) 2 points: Which of the following statements is not a correct finite difference approximation to $\frac{dV}{dx}$ at x_0 if $h = \Delta x$? Name the others.

   a) $\frac{V(x_0+h)-V(x_0)}{h}$

   b) $\frac{V(x_0)-V(x_0-h)}{h}$

   c) $\frac{V(x_0+h)-V(x_0-h)}{h}$

   d) $\frac{V(x_0+h/2)-V(x_0-h/2)}{h}$

   e) $\frac{V(x_0+h)-V(x_0-h)}{2h}$

b) 2 points: Using the difference equation $V_n = V_{n-1} + V_{n+1}$ with $V_0 = V_5 = 1$ and starting with initial values $V_n = 0$ for $1 \leq n \leq 4$, the value of V_2 after the fourth iteration is:

   a) 1 b) 3 c) 4 d) 9 e) 25

c) 1 point: The triangular element of Figure 3 is in free space. The approximate value of the potential at the center of the triangle is

   a) 10 V b) 7.5 V c) 5 V d) 2.5 V

d) 2 point: The area of the element in Figure 3 is:

   a) 14 b) 8 c) 7 d) 4 e) 3

e) 1 point:1 Which of these statements is not true about finite element shape functions?

   a) They are interpolatory in nature.

   b) They must be continuous across the elements.

   c) Their sum is identically equal to unity at every point within the element.

   d) The shape function associated with a given node vanishes at any other node.

   e) The shape function associated with a node is zero at that node.

f) 1 point: A major difference between the finite difference and the finite element methods is that:

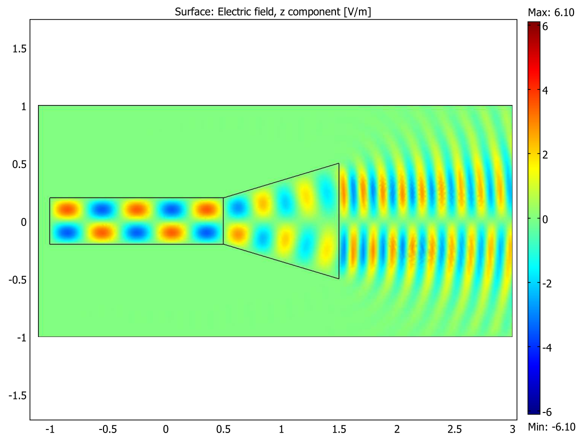
   a) Using one, a sparse matrix results in the solution.

   b) In one, the solution is known at all points in the domain.

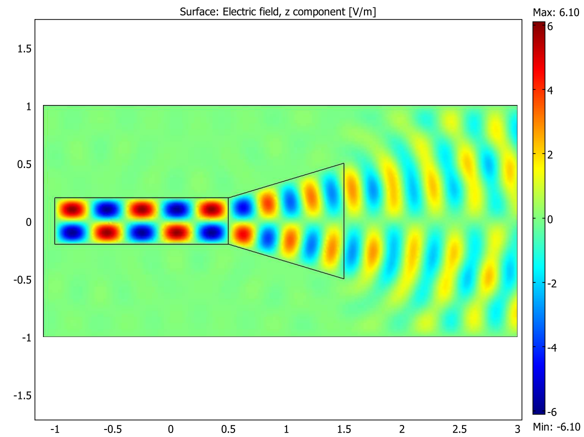
   c) One applies to solving partial differential equation.

   d) One is limited to time-invariant problems.

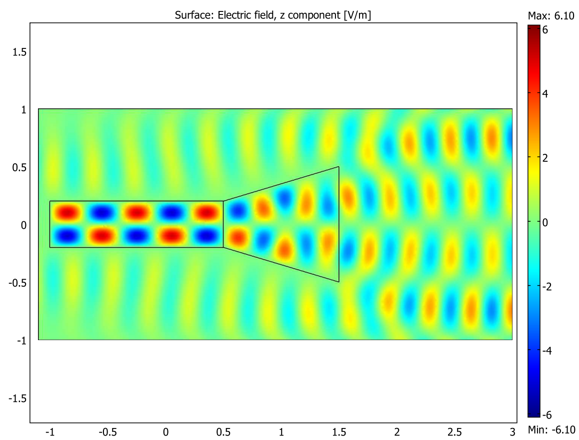
g) 4 point: State two physical problems, which can be described by the Poisson equation, and two physical problems for the Laplace equation. Total: 14 points



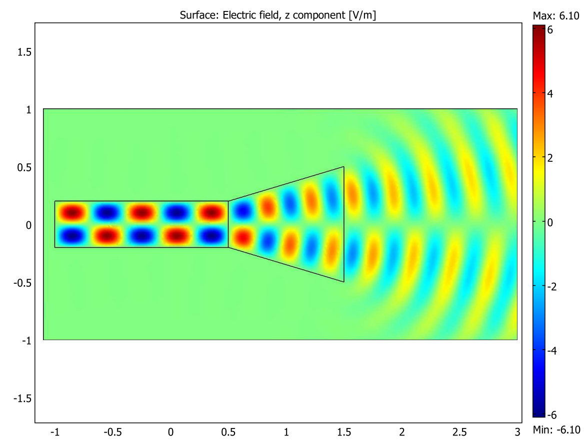
(a) Simulation 1



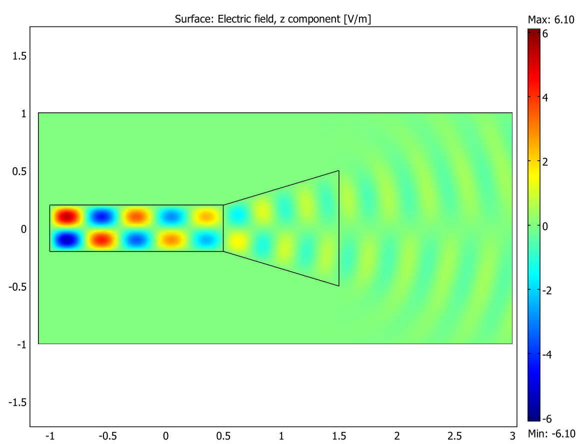
(b) Simulation 2



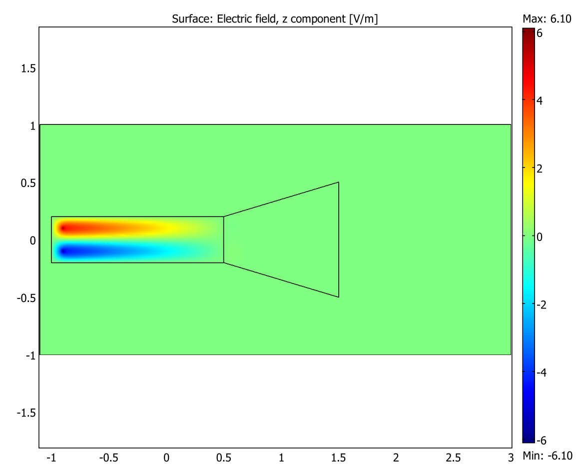
(c) Simulation 3



(d) Simulation 4



(e) Simulation 5



(f) Simulation 6

Figure 2: Comsol Multiphysics simulation results in the xy plane. All simulation results depicted with the same phase. (Scale in meter and V/m)

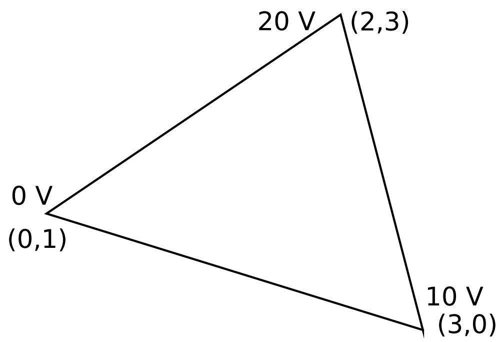


Figure 3: triangle in free space