1) **Solution**

1. The wall is infinite, therefore, the potential ϕ on it is zero. Let us introduce a cylindrical coordinate system (ρ, θ, z), as shown in Fig. 1.a. The task is symmetric with respect to θ.

   We can replace the PEC wall by a mirror charge -q, placed on the opposite side of the wall, at distance -h. Using the principal of superposition, the potential at point (ρ, z) is equal to:

$$\phi(\rho, z) = q\left(\frac{1}{\sqrt{\rho^2 + (z-h)^2}} - \frac{1}{\sqrt{\rho^2 + (z+h)^2}}\right) \tag{1}$$

   The potential on the wall $\phi(\rho, z=0)$ is thus zero.
   Electric field $\vec{E}(\vec{r})$ is given by the field superposition of two charges:

$$\vec{E}(\vec{r}) = -\frac{q}{|\vec{r}_2|^3}\vec{r}_2 + \frac{q}{|\vec{r}_1|^3}\vec{r}_1 \tag{2}$$

   Distribution of charge density $\sigma(\rho, z=0)$ is found from:

$$\sigma(\rho) = -\frac{1}{4\pi}\frac{\partial \phi}{\partial z}|_{z=0} = -\frac{1}{2\pi}\frac{qh}{(\rho^2+h^2)^{3/2}} \tag{3}$$

   Total charge induced on the wall is the integral of charge density over the whole surface:

$$Q = \int_0^{+\infty}\int_0^{2\pi} \sigma(\rho)\rho d\rho d\theta = -q \tag{4}$$

   We can see that the charge +q induces charge -q on the PEC wall.

2. Mirror charges should be placed as shown in Fig. 1.b. It can be shown that the potential on a corner wall is equal to zero.
   The distribution of the electric field is given by field superposition of four charges:

$$\vec{E}(\vec{r}) = -\frac{q}{|\vec{r}_1|^3}\vec{r}_1 + \frac{q}{|\vec{r}_2|^3}\vec{r}_2 - \frac{q}{|\vec{r}_3|^3}\vec{r}_3 + \frac{q}{|\vec{r}_4|^3}\vec{r}_4 \tag{5}$$

   Distribution of charge density $\sigma(\rho, z=0) = \sigma(\rho=0, z)$ is:

$$\sigma(\rho) = -\frac{1}{4\pi}\frac{\partial \phi}{\partial z}|_{z=0} = -\frac{q}{4\pi}\left(\frac{2h}{((\rho-h)^2+h^2)^{3/2}} - \frac{2h}{((\rho+h)^2+h^2)^{3/2}}\right) \tag{6}$$

3. We solve this task in two steps. First, consider that the sphere is grounded, so that the potential is zero on the surface $\phi(R, \theta) = 0$. Here we introduced spherical coordinates (r, θ, φ) as shown in, Fig. 1.c. We have symmetry with respect to angle φ. We place a mirror charge $q' = -qR/h$ at distance $a = R^2/h$ from the center of sphere. One can check that condition $\phi(R, \theta) = 0$ is thus fulfilled. Once the sphere is grounded, a total charge q' is induced on its surface. Next, we unplug the sphere from the ground. Since the sphere was neutral initially, we should add a charge $-q'$ to its surface. This charge $-q'$ will be distributed uniformly on the surface, since the electrostatic forces are zero on the sphere. In terms of potential, adding a charge $-q'$, is equivalent to putting a charge $-q'$ in the center of the sphere. So the total potential outside the sphere is given by superposition of the potentials of three charges:

$$\phi(r, \theta) = q\frac{1}{|\vec{r}_1|} + q'\frac{1}{|\vec{r}_2|} - q'\frac{1}{|\vec{r}_3|} \tag{7}$$

   Potential on the surface of the sphere is:

$$\phi(R, \theta) = \frac{q}{\sqrt{R^2+h^2-2Rh\cos\theta}} + \frac{q'}{\sqrt{R^2+a^2-2Ra\cos\theta}} - \frac{q'}{R} \tag{8}$$

   Distribution of charge density on the surface $\sigma(\theta)$ is:

$$\sigma(\theta) = -\frac{1}{4\pi}\frac{\partial \phi}{\partial r}|_{r=R} = -\frac{1}{4\pi}\left(\frac{q(h^2-R^2)}{R(R^2+h^2-2Rh\cos\theta)^{3/2}} - \frac{q}{Rh}\right) \tag{9}$$

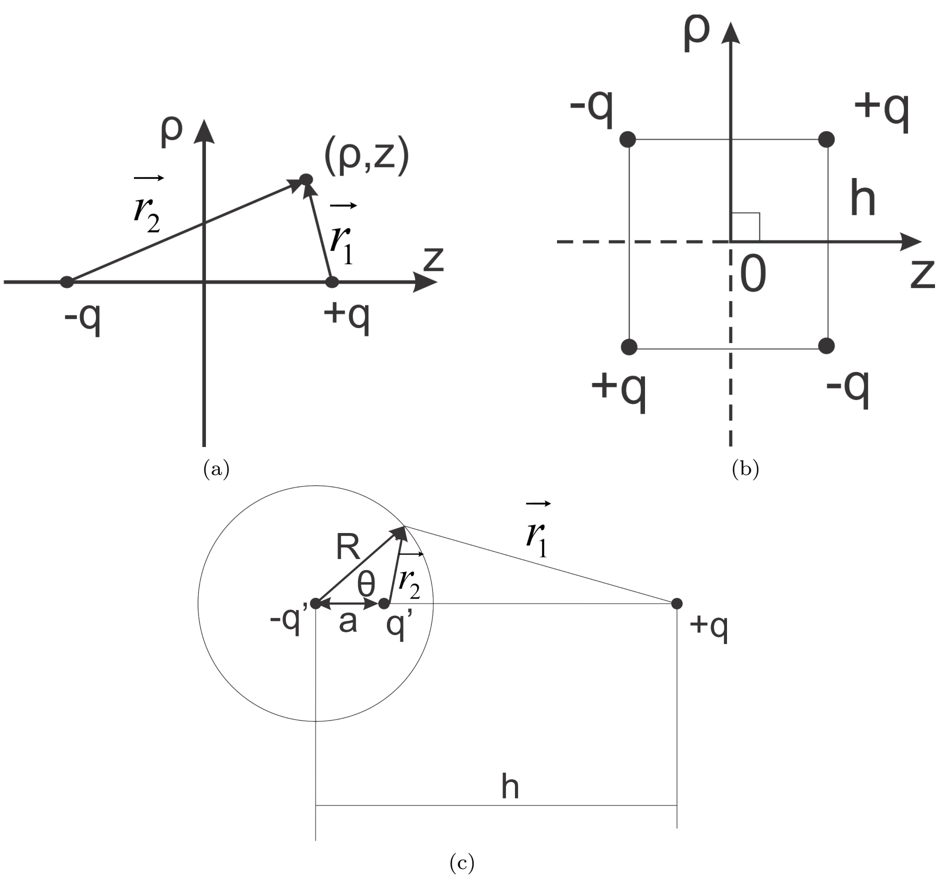

Figure 1