Exercise 5

In free space ($z \leq 0$) a plane wave described by

$$\vec{H}_i = 10\cos(10^8 t - \beta z)\vec{e}_x \, \frac{\text{mA}}{\text{m}} \tag{1}$$

is incident normally on a lossless medium ($\varepsilon_r = 2$, $\mu_r = 8$ for $z \geq 0$) as shown in fig. 1. Determine the reflected wave $\vec{H}_r$, $\vec{E}_r$ and the transmitted wave $\vec{H}_t$, $\vec{E}_t$.

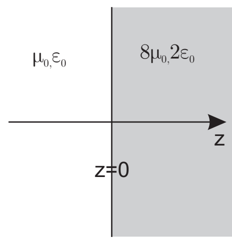

Figure 1: Plane wave incident normally on medium

Exercise 6

A parallel plate waveguide, see fig. 2, is illuminated by two plane waves. The superposition of the two linear polarized plane waves is used to represent the field inside the waveguide

$$\vec{E}_1 = E e^{j\omega t - j\beta_x x - j\beta_z z}\vec{e}_y \tag{2}$$

$$\vec{E}_2 = -E e^{j\omega t + j\beta_x x - j\beta_z z}\vec{e}_y. \tag{3}$$

a) Compute the complete field in the waveguide.

b) Determine β_x using the boundary conditions.

c) Determine β_z using the relation for plane waves $\beta_x^2 + \beta_z^2 = \omega^2\mu\epsilon$.

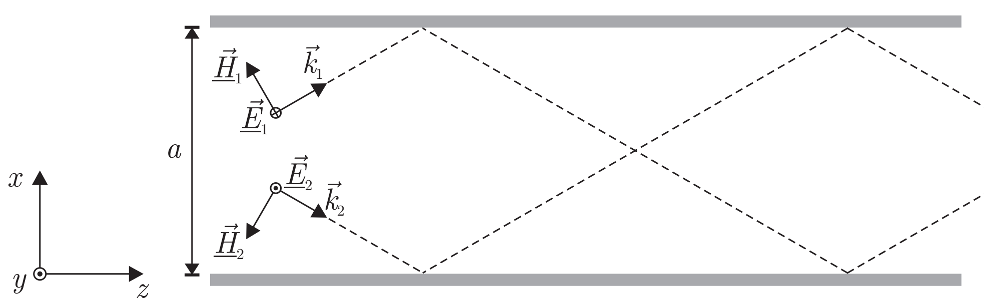

Figure 2: Parallel plate waveguide

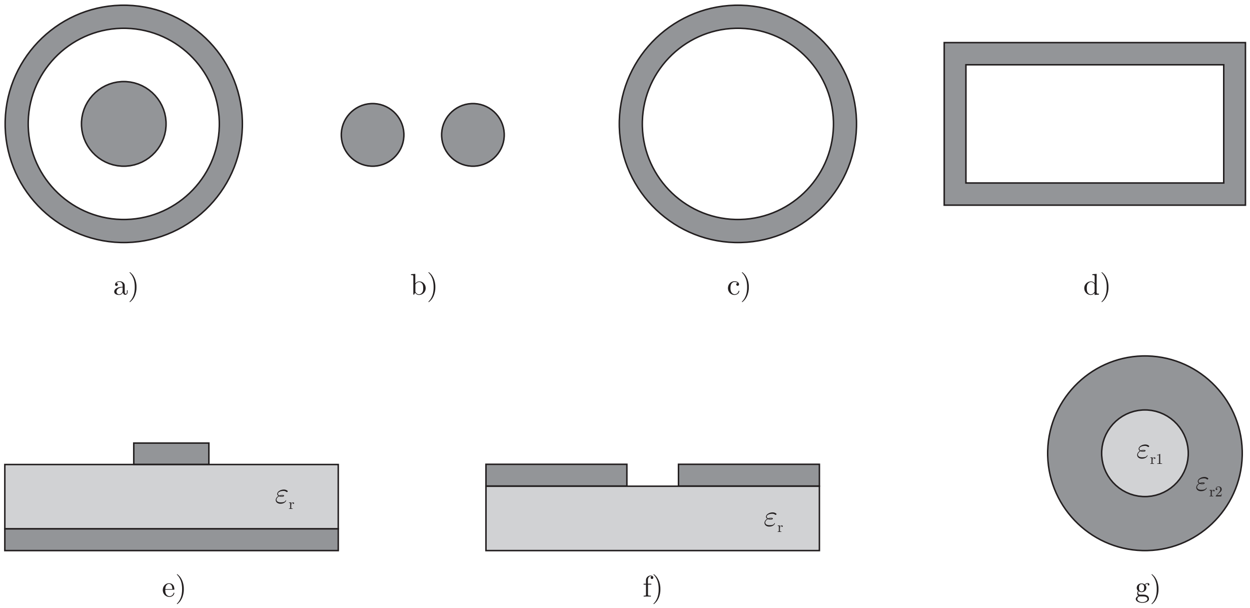

Figure 3: (a) Coaxial line (b) Two wire line (c) Circular Waveguide (d) Rectangular waveguide (e) Microstripline (f) Slotline (g) Optical fiber

Appendix

Waveguides in 3D

Some technical available versions of waveguides are depicted in fig. 3. All waveguides have in common that their geometry does not depend on z-space. The field solution of a specific waveguide can be written in general for the harmonic case as

$$\vec{E}(x,y,z,t) = \vec{E}(x,y)e^{j\omega t-\gamma z} \quad (4)$$
$$\vec{H}(x,y,z,t) = \vec{H}(x,y)e^{j\omega t-\gamma z}. \quad (5)$$

Utilising the governing differential equations, i.e. Maxwell's equations, for a specific waveguide problem and incorporating its boundary conditions given by its geometry as well as its material parameters the following quantities can be determined:

$$\vec{E}(x,y), \vec{H}(x,y), \gamma. \quad (6)$$

Thus for different geometries, different solutions are obtained, which are characterised by special special properties like:

$E_z = 0$	$H_z = 0$	TEM modes
$E_z = 0$	$H_z \neq 0$	TE or H modes
$E_z \neq 0$	$H_z = 0$	TM or E modes
$E_z \neq 0$	$H_z \neq 0$	Hybrid modes

Concerning γ, the solution process yields in general in

$$k_c^2 = \omega^2 \epsilon \mu - \gamma^2, \tag{7}$$

where k_c is a complex number which is called cutoff wavenumber. Since the propagation constant β is zero at the cutoff frequency ω_c, (7) results in

$$\gamma = \alpha + j\beta = \sqrt{\omega_c^2 \mu \epsilon - k_c^2}. \tag{8}$$

Waveguides in 2D

A waveguide in 2D requires that the field in one direction, e.g. $\vec{e_y}$, is uniform, hence the solution yield to

$$\vec{E}(x,y,z,t) = \vec{E}(x) e^{j\omega t - \gamma z} \tag{9}$$

$$\vec{H}(x,y,z,t) = \vec{H}(x) e^{j\omega t - \gamma z}. \tag{10}$$

Instead of solving Maxwell's equations and using boundary conditions to solve for unknowns, a different approach can be used. Maxwell's equation and the assumptions in ((9)) and ((10)) are already fulfilled by the plane wave solution with propagation direction in the x, z-plane. Its boundary conditions given by geometry and material parameters of the 2D waveguide are used to determine propagation direction, amplitude and polarization of each plane wave in a sum of plane waves, which constitutes the solution of the field problem. To substantiate ideas see Exercise 6.