Exercise 8

A string of length $l = 1$ m is connected at both ends to a wall and is subjected to an external force per unit length of $F(x) = 1(x)\,\frac{\text{MN}}{\text{m}}$ at $0.29 \leq x \leq 0.31$ m and $F(x) = 0\,\frac{\text{MN}}{\text{m}}$ elsewhere. Find the displacement $u(x)$ of the string assumed that the load $F(x)$ is acting down (negative direction) and the displacement $u(x)$ is governed by the differential equation

$$T\frac{d^2u}{dx^2} = -F(x)$$

where $T = 70$ MPa (copper) is the uniform tensile force of the string.

a) What are the boundary conditions for this problem?

b) Derive a finite element scheme for this problem using Galerkin's method.

c) Which simple shape function would be appropriate?

d) Write a 1D computer code to evaluate the displacement $u(x)$.

e) Solve the problem by Comsol in 1D and compare the results.