Introduction to BEM

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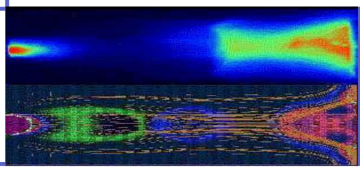

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Outline

- Introduction to the basic idea of BEM
- Integral formulation of 3D Laplace problem
- BEM treatment of 3D Laplace problem
- 3D electrostatic analysis example
- Matrix compression (Fast multiple technique)
- Singular integrals
- Integral formulation of 3D eddy-currents problem
- 3D eddy-currents analysis example
- Conclusions

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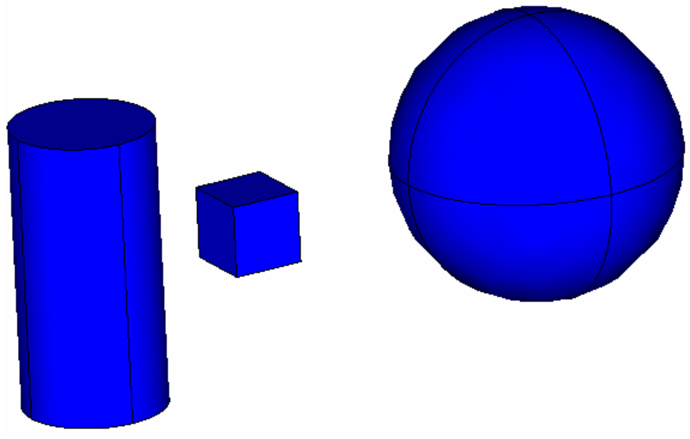
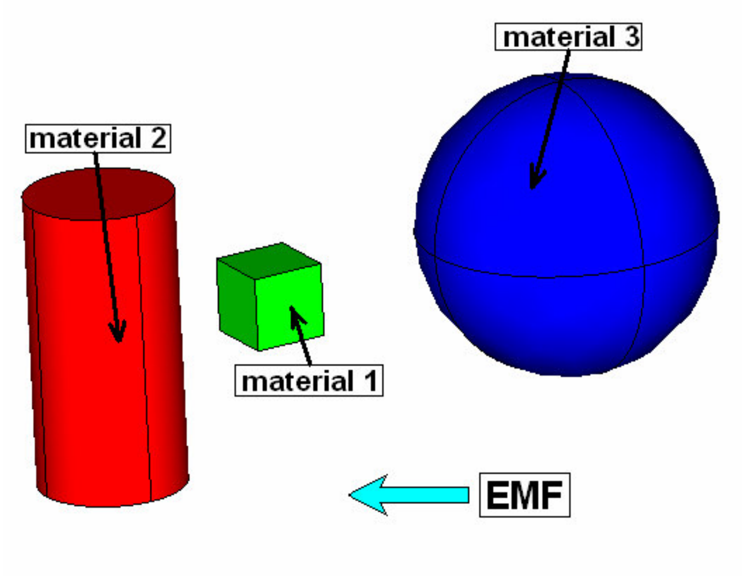
Introduction to the basic idea of BEM





Mathematical description

$$\left(I\vec{f}\right)(\vec{x}) = 0,\ \vec{x} \in \partial\Omega$$

Model parameters

$$material : (\mu, \sigma, \varepsilon)$$

$$frequency : \quad (f)$$

$$sources : (\vec{E}_s, \vec{H}_s, \vec{J}_s)$$

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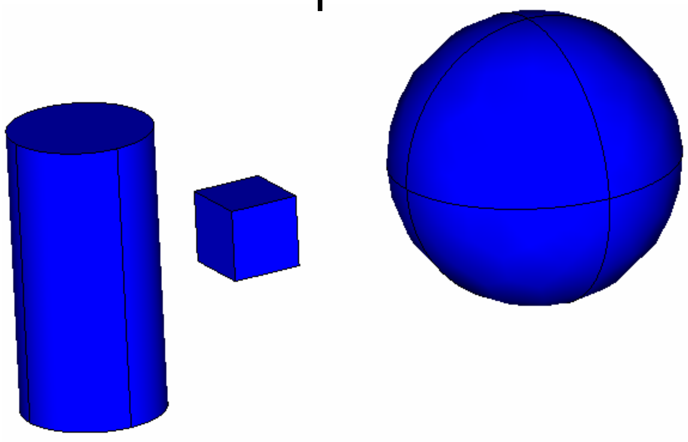
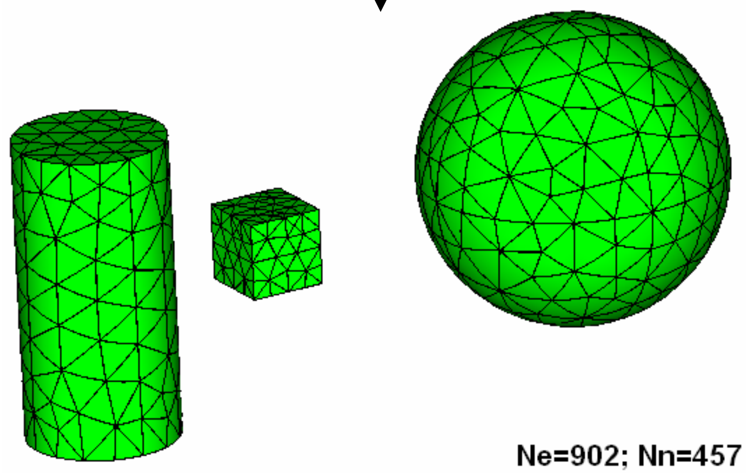
Introduction to the basic idea of BEM

Model discretization





Integral formulation

$$\left(I\vec{f}\right)(\vec{x}) = 0,\ \vec{x} \in \partial\Omega$$

→ **BEM scheme** →

$$[A]\{x\} = \{b\}$$

Large dense linear system of equations

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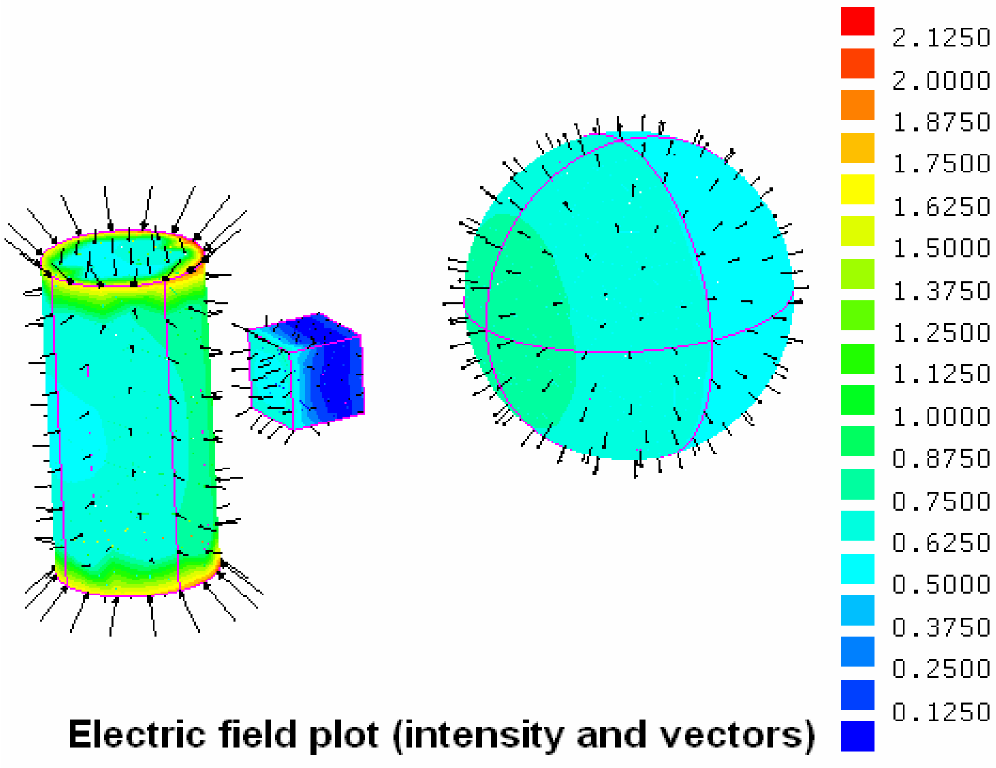
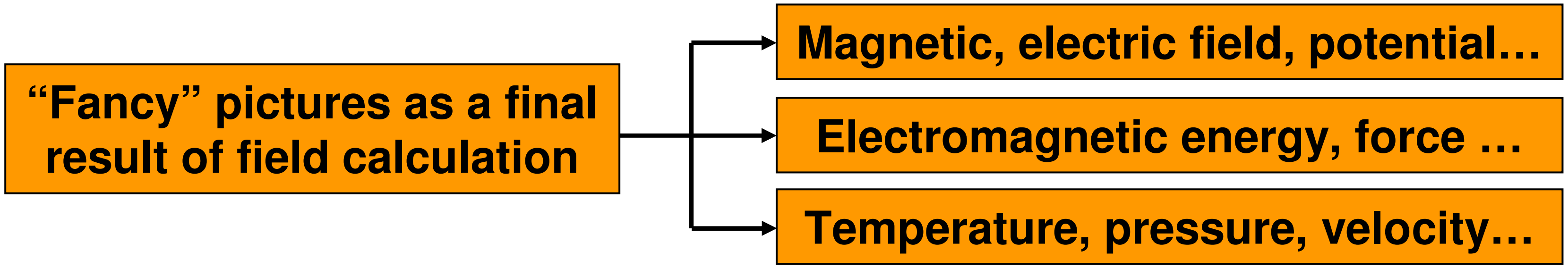
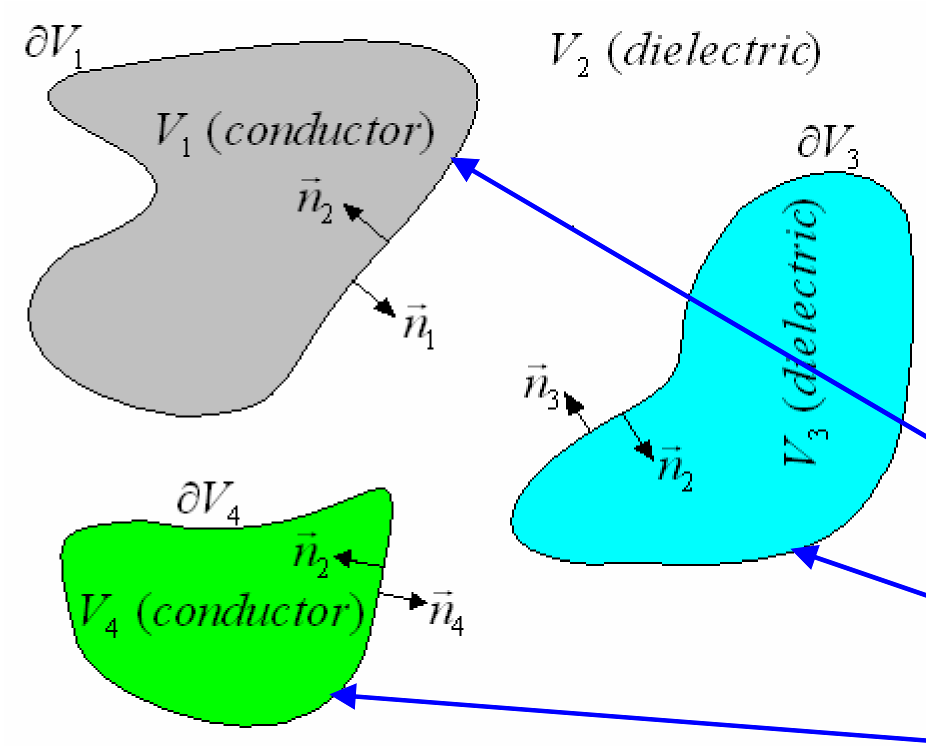


Electric field plot (intensity and vectors)



Integral formulation of 3D Laplace problem



Materials:

1. **Perfect electric conductors (fixed potential electrode -FIXE)**
2. **Perfect electric conductors (floating potential electrode - FLOE)**
3. **Linear homogenous dielectrics**

$$\Delta_y \varphi_k(y) = 0,\ y \in V_k \subseteq R^3$$

$$\varphi_1(x) = \varphi_2(x) = V_0,\ x \in \partial V_1$$

$$\varepsilon_2 \frac{\partial \varphi_2}{\partial n_2}(x) = -\varepsilon_3 \frac{\partial \varphi_3}{\partial n_3}(x),\ x \in \partial V_3$$

$$\nabla \varphi_2(x) \times \vec{n}_4 = 0,\ x \in \partial V_4$$

3D electrostatic analysis

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Integral formulation of 3D Laplace problem

$$\Delta_y \varphi_k(y) = 0,\ y \in V_k \subseteq R^3$$

$$-\Delta_y G(x,y) = \delta(x,y),\ y \in V_k \subseteq R^3,\ x \in R^3$$

$$G(x,y) = \frac{1}{4\pi}\frac{1}{|x-y|}$$

Green's function

$$G(x,y)\cdot\Delta_y\varphi_k(y) - \varphi_k(y)\cdot\Delta_y G(x,y) = \delta(x,y)\cdot\varphi_k(y) \ \Big|\ \iiint_{(V_k)}$$

$$\iiint_{(V_k)}\left[G(x,y)\cdot\Delta_y\varphi_k(y) - \varphi_k(y)\cdot\Delta_y G(x,y)\right]dV = \iiint_{(V_k)}\delta(x,y)\cdot\varphi_k(y)dV$$

$$\iiint_{(V_k)}\delta(x,y)\cdot\varphi_k(y)dV = \frac{\theta(x)}{4\pi}\varphi_k(x)$$

$$\theta(x) = \begin{cases} 4\pi,\ x \in V_k - \partial V_k \\ 2\pi,\ x \in \partial V_k,\ if\ \partial V_k\ is\ smooth \\ 0,\ x \notin V_k \end{cases}$$

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Integral formulation of 3D Laplace problem

$$\iiint_{(V_k)} \left\{ \nabla_y \left[G(x,y) \cdot \nabla_y \varphi_k(y) \right] - \nabla_y \left[\varphi_k(y) \cdot \nabla_y G(x,y) \right] \right\} dV = \frac{\theta(x)}{4\pi} \varphi_k(x)$$

Volume integration

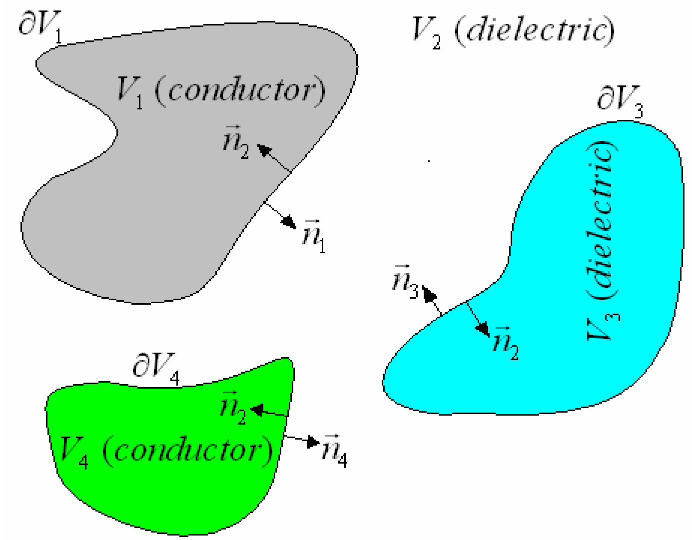
$(Green's\ theorem) \Rightarrow$

$$\oiint_{\partial V_k} \left[G(x,y) \cdot \nabla_y \varphi_k(y) \right] \cdot \vec{n}_y dS_y - \oiint_{(\partial V_k)} \left[\varphi_k(y) \cdot \nabla_y G(x,y) \right] \cdot \vec{n}_y dS_y = \frac{\theta(x)}{4\pi} \varphi_k(x)$$

$$\oiint_{\partial V_k} G(x,y) \cdot \frac{\partial \varphi_k}{\partial n_y}(y)\, dS_y - \oiint_{(\partial V_k)} \varphi_k(y) \cdot \frac{\partial G}{\partial n_y}(x,y)\, dS_y = \frac{\theta(x)}{4\pi} \varphi_k(x)$$

Integration over boundary

Integral formulation of 3D Laplace problem



$x \in R^3$

$$\oiint_{(\partial V_1)} G(x,y)\cdot\frac{\partial\varphi_1}{\partial n_{1y}}(y)\,dS_y - \oiint_{(\partial V_1)} \varphi_1(y)\cdot\frac{\partial G}{\partial n_{1y}}(x,y)\,dS_y = \frac{\theta_1(x)}{4\pi}\varphi_1(x)$$

$$\oiint_{(\partial V_2)} G(x,y)\cdot\frac{\partial\varphi_2}{\partial n_{2y}}(y)\,dS_y - \oiint_{(\partial V_2)} \varphi_2(y)\cdot\frac{\partial G}{\partial n_{2y}}(x,y)\,dS_y = \frac{\theta_2(x)}{4\pi}\varphi_2(x)$$

$$\oiint_{(\partial V_3)} G(x,y)\cdot\frac{\partial\varphi_3}{\partial n_{3y}}(y)\,dS_y - \oiint_{(\partial V_3)} \varphi_3(y)\cdot\frac{\partial G}{\partial n_{3y}}(x,y)\,dS_y = \frac{\theta_3(x)}{4\pi}\varphi_3(x)$$

$$\oiint_{(\partial V_4)} G(x,y)\cdot\frac{\partial\varphi_4}{\partial n_{4y}}(y)\,dS_y - \oiint_{(\partial V_4)} \varphi_4(y)\cdot\frac{\partial G}{\partial n_{4y}}(x,y)\,dS_y = \frac{\theta_4(x)}{4\pi}\varphi_4(x)$$

Integral formulation of 3D Laplace problem

$$\oiint_{(\partial V_1)} G(x,y)\cdot\left[\frac{\partial \varphi_1}{\partial n_{1y}}(y)-\frac{\partial \varphi_2}{\partial n_{1y}}(y)\right]dS_y - \oiint_{(\partial V_1)}[\varphi_1(y)-\varphi_2(y)]\cdot\frac{\partial G}{\partial n_{1y}}(x,y)\,dS_y +$$
$$+\oiint_{(\partial V_3)} G(x,y)\cdot\left[\frac{\partial \varphi_3}{\partial n_{3y}}(y)-\frac{\partial \varphi_2}{\partial n_{3y}}(y)\right]dS_y - \oiint_{(\partial V_3)}[\varphi_3(y)-\varphi_2(y)]\cdot\frac{\partial G}{\partial n_{3y}}(x,y)\,dS_y +$$
$$+\oiint_{(\partial V_4)} G(x,y)\cdot\left[\frac{\partial \varphi_4}{\partial n_{4y}}(y)-\frac{\partial \varphi_2}{\partial n_{4y}}(y)\right]dS_y - \oiint_{(\partial V_4)}[\varphi_4(y)-\varphi_2(y)]\cdot\frac{\partial G}{\partial n_{4y}}(x,y)\,dS_y$$
$$=\frac{\theta_1(x)}{2\pi}\varphi_1(x)+\frac{\theta_2(x)}{2\pi}\varphi_2(x)+\frac{\theta_3(x)}{2\pi}\varphi_3(x)+\frac{\theta_4(x)}{2\pi}\varphi_4(x)$$

↓

$$-\oiint_{(\partial V_1)} G(x,y)\cdot\frac{\partial \varphi_2}{\partial n_{1y}}(y)\,dS_y + \oiint_{(\partial V_3)} G(x,y)\cdot\left[\frac{\partial \varphi_3}{\partial n_{3y}}(y)-\frac{\partial \varphi_2}{\partial n_{3y}}(y)\right]dS_y +$$
$$-\oiint_{(\partial V_4)} G(x,y)\cdot\frac{\partial \varphi_2}{\partial n_{4y}}(y)\,dS_y = \frac{\theta_1(x)}{2\pi}\varphi_1(x)+\frac{\theta_2(x)}{2\pi}\varphi_2(x)+\frac{\theta_3(x)}{2\pi}\varphi_3(x)+\frac{\theta_4(x)}{2\pi}\varphi_4(x)$$

(*)

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Integral formulation of 3D Laplace problem

BEM is related to the field sources over the boundary

$$y \in \partial V_1 \Rightarrow \varepsilon_2 \frac{\partial \varphi_2}{\partial n_{1y}}(y) = -\sigma_{1f}(y), \sigma_1'(y) = \sigma_{1f}(y) + \sigma_{1pol}(y) = -\varepsilon_0 \frac{\partial \varphi_2}{\partial n_{1y}}(y), \sigma_1(y) = \frac{\sigma_1'(y)}{\varepsilon_0}$$

$$y \in \partial V_3 \Rightarrow \frac{\partial \varphi_3}{\partial n_{3y}}(y) - \frac{\partial \varphi_2}{\partial n_{3y}}(y) = \left(1 - \frac{\varepsilon_2}{\varepsilon_3}\right) \frac{\partial \varphi_2}{\partial n_{3y}}(y) = \frac{\sigma_3'(y)}{\varepsilon_0}, \sigma_3(y) = \frac{\sigma_3'(y)}{\varepsilon_0}$$

$$y \in \partial V_4 \Rightarrow \varepsilon_2 \frac{\partial \varphi_2}{\partial n_{4y}}(y) = -\sigma_{4f}(y), \sigma_4'(y) = \sigma_{1f}(y) + \sigma_{1pol}(y) = -\varepsilon_0 \frac{\partial \varphi_2}{\partial n_{4y}}(y), \sigma_4(y) = \frac{\sigma_4'(y)}{\varepsilon_0}$$

Our integral formulation deals with a total surface charge density !

Total charge = Free charge (conductor) + Polarized charge (dialect. interface)

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Integral formulation of 3D Laplace problem

$$x \in \partial V_1 \stackrel{(*)}{\Rightarrow}$$

$$\oiint_{(\partial V_1)} G(x,y) \cdot \sigma_1(y)\, dS_y + \oiint_{(\partial V_3)} G(x,y) \cdot \sigma_3(y)\, dS_y + \oiint_{(\partial V_4)} G(x,y) \cdot \sigma_4(y)\, dS_y = V_0$$

OK!

$$x \in \partial V_3 \stackrel{(*)}{\Rightarrow}$$

$$\oiint_{(\partial V_1)} G(x,y) \cdot \sigma_1(y)\, dS_y + \oiint_{(\partial V_3)} G(x,y) \cdot \sigma_3(y)\, dS_y + \oiint_{(\partial V_4)} G(x,y) \cdot \sigma_4(y)\, dS_y = \frac{\theta_2(x)}{4\pi}\varphi_2(x) + \frac{\theta_3(x)}{4\pi}\varphi_3(x)$$

?!!

Problem: potential and charge density are unknowns (in the same time)!

Solution:

$$\oiint_{(\partial V_1)} G(x,y) \cdot \sigma_1(y)\, dS_y + \oiint_{(\partial V_3)} G(x,y) \cdot \sigma_3(y)\, dS_y + \oiint_{(\partial V_4)} G(x,y) \cdot \sigma_4(y)\, dS_y = \frac{\theta_2(x)}{4\pi}\varphi_2(x) + \frac{\theta_3(x)}{4\pi}\varphi_3(x) \;\Big|\; \frac{\partial}{\partial n_{3x}}$$

Integral formulation of 3D Laplace problem

$x \in \partial V_3 \Rightarrow$

Dielectrics interface

$$\oiint_{(\partial V_1)} \frac{\partial G}{\partial n_{3x}}(x,y)\cdot\sigma_1(y)\,dS_y + \oiint_{(\partial V_3)} \frac{\partial G}{\partial n_{3x}}(x,y)\cdot\sigma_3(y)\,dS_y + \oiint_{(\partial V_4)} \frac{\partial G}{\partial n_{3x}}(x,y)\cdot\sigma_4(y)\,dS_y =$$

$$= \frac{1}{2}\frac{\varepsilon_3+\varepsilon_2}{\varepsilon_3-\varepsilon_2}\sigma_3(x)$$

$x \in \partial V_4 \Rightarrow$

Floating potential electrode

$$\oiint_{(\partial V_1)} \frac{\partial G}{\partial n_{4x}}(x,y)\cdot\sigma_1(y)\,dS_y + \oiint_{(\partial V_3)} \frac{\partial G}{\partial n_{4x}}(x,y)\cdot\sigma_3(y)\,dS_y + \oiint_{(\partial V_4)} \frac{\partial G}{\partial n_{4x}}(x,y)\cdot\sigma_4(y)\,dS_y =$$

$$= -\frac{1}{2}\sigma_4(x)$$

$x \in \partial V_1 \Rightarrow$

Fixed potential electrode

$$\oiint_{(\partial V_1)} G(x,y)\cdot\sigma_1(y)\,dS_y + \oiint_{(\partial V_3)} G(x,y)\cdot\sigma_3(y)\,dS_y + \oiint_{(\partial V_4)} G(x,y)\cdot\sigma_4(y)\,dS_y = V_0$$

BEM treatment of 3D Laplace problem

Integral formulation → $(I\,\sigma)(x) = h(x),\ x \in \partial\Omega$

$(I\,\sigma)(x) = h(x),\ x \in \partial\Omega$ → **BEM discretization** → $[A]\{\sigma\} = \{b\}$

Large dense linear system of equations → $[A]\{\sigma\} = \{b\}$

BEM discretization

- **Weighted residual method** → $\iint\limits_{(\partial\Omega)} w_i(x)\cdot[(I\,\sigma)(x) - h(x)]\cdot dS_x = 0,\ i = 1,2,3,\ldots,N$
  - **Point matching (collocation)** → $w_i(x) = \delta(x_i, x)$
  - **Galerkin** → $w_i(x) = N_i(x)$

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BEM treatment of 3D Laplace problem

Point matching (collocation) → $w_i(x) = \delta(x_i, x)$

$$(I\,\sigma)(x_i) = h(x_i);\ i = 1,2,3,\ldots, N;\ x_i \in \partial\Omega$$

Linear nodal triangular element →

$$\sigma^e(x) = \sum_{j=1}^{3} N_j^e(x) \cdot \sigma_j^e,\ x \in \Delta^e$$

Concept of elemental contribution

$$(F\sigma)(x_i) = (I\,\sigma)(x_i) - h(x_i)$$

$$(F\sigma)(x) = \sum_{e=1}^{N_e} \left(F^e \sigma^e\right)(x) = \sum_{e=1}^{N^e} \left([K^e]\{\sigma^e\} - \{b^e\}\right)$$

$$K_{ij}^e = \begin{cases} \iint_{(\Delta^e)} \left(G(x_i, y) \cdot N_j^e(y)\right) d\Omega,\ \Delta^e \in \partial V_1 \\ C + \iint_{(\Delta^e)} \left(\dfrac{\partial G}{\partial n_y}(x_i, y) \cdot N_j^e(y)\right) d\Omega,\ \Delta^e \in \partial V_{3,4} \end{cases}$$

$$b_i^e = \begin{cases} V_0,\ x_i \in \partial V_1 \\ 0,\ otherwise \end{cases}$$

$$i, j = 1,2,3$$

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BEM treatment of 3D Laplace problem

Large dense linear system of equations → $[A]\{\sigma\}=\{b\}$

- Iterative solver has to be used
- BiCG, GMRES
- Asymmetric matrix has been produced
- Preconditioning of the matrix is very important
- Matrix compression is necessary
- Fast multipole algorithm (improvement for matrix compression and solution speed)

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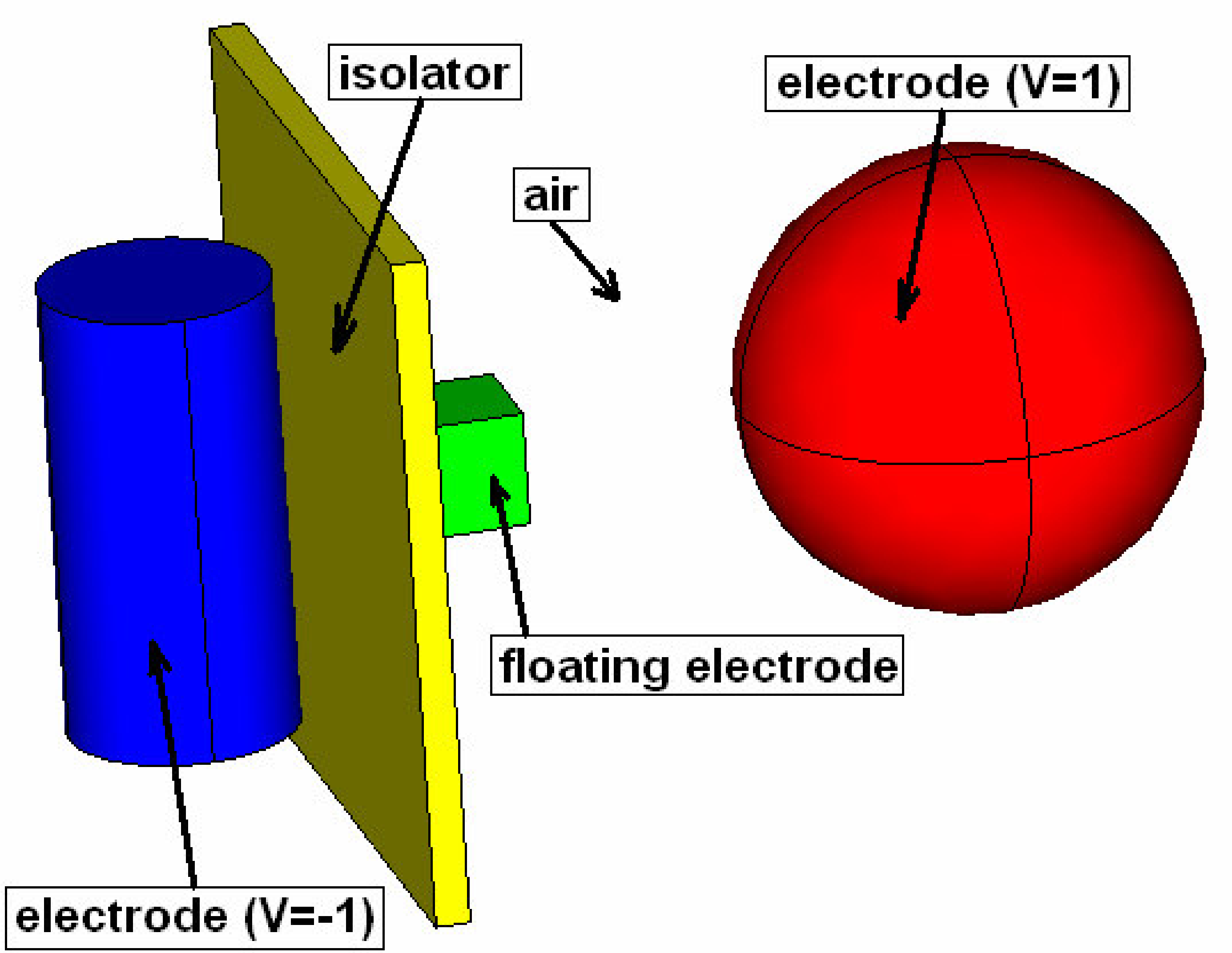
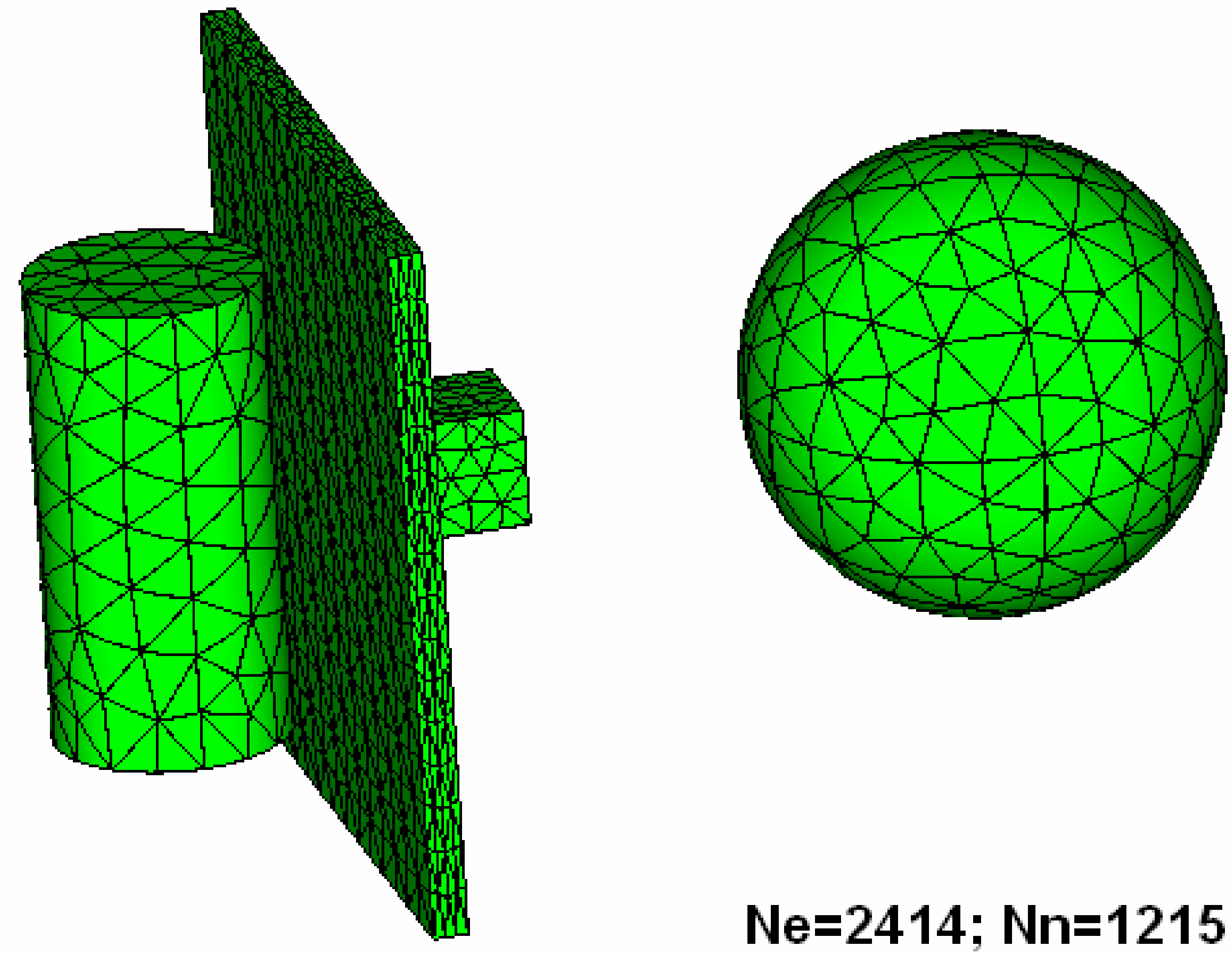
3D electrostatic example

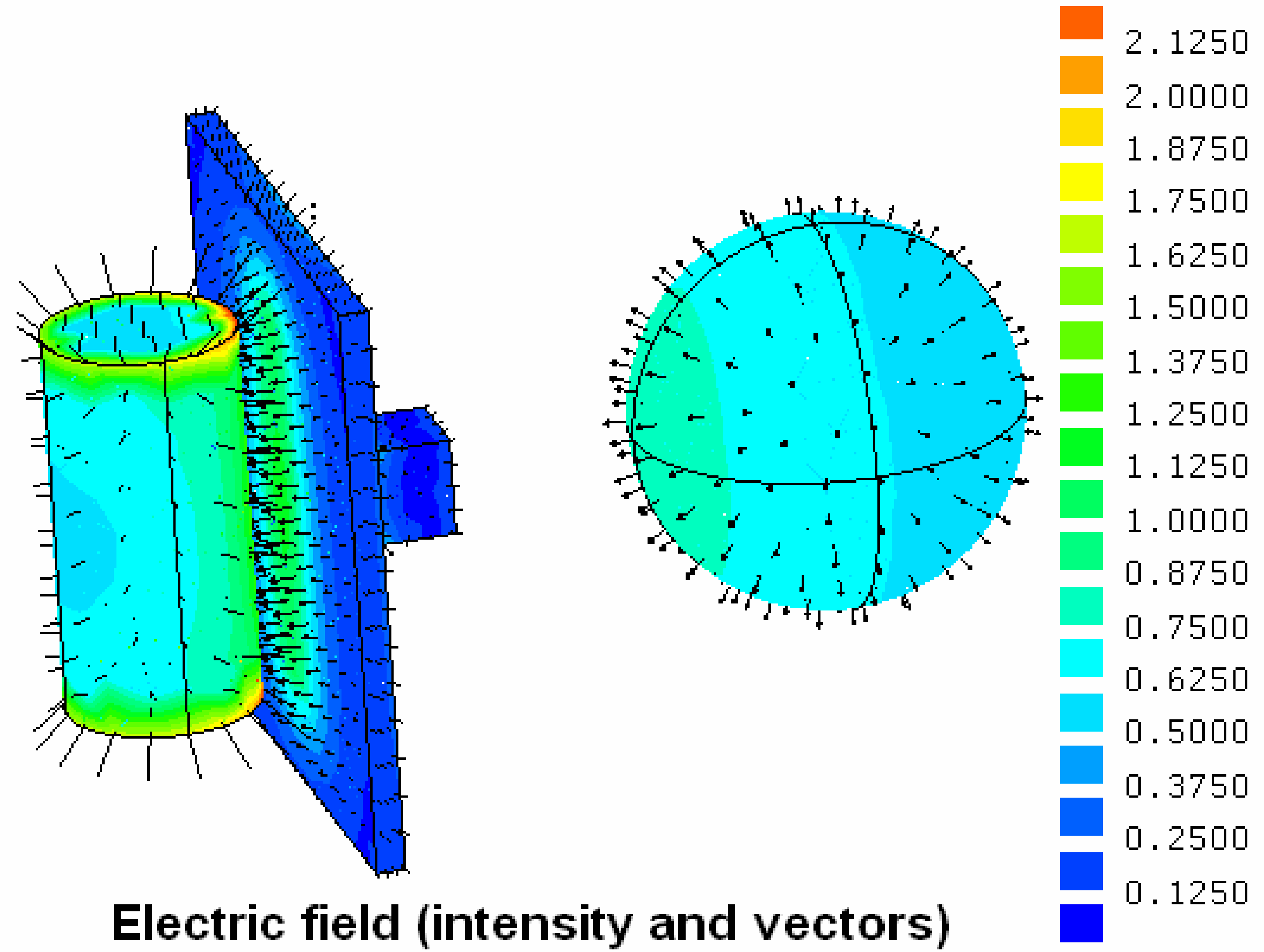


3D electrostatic example



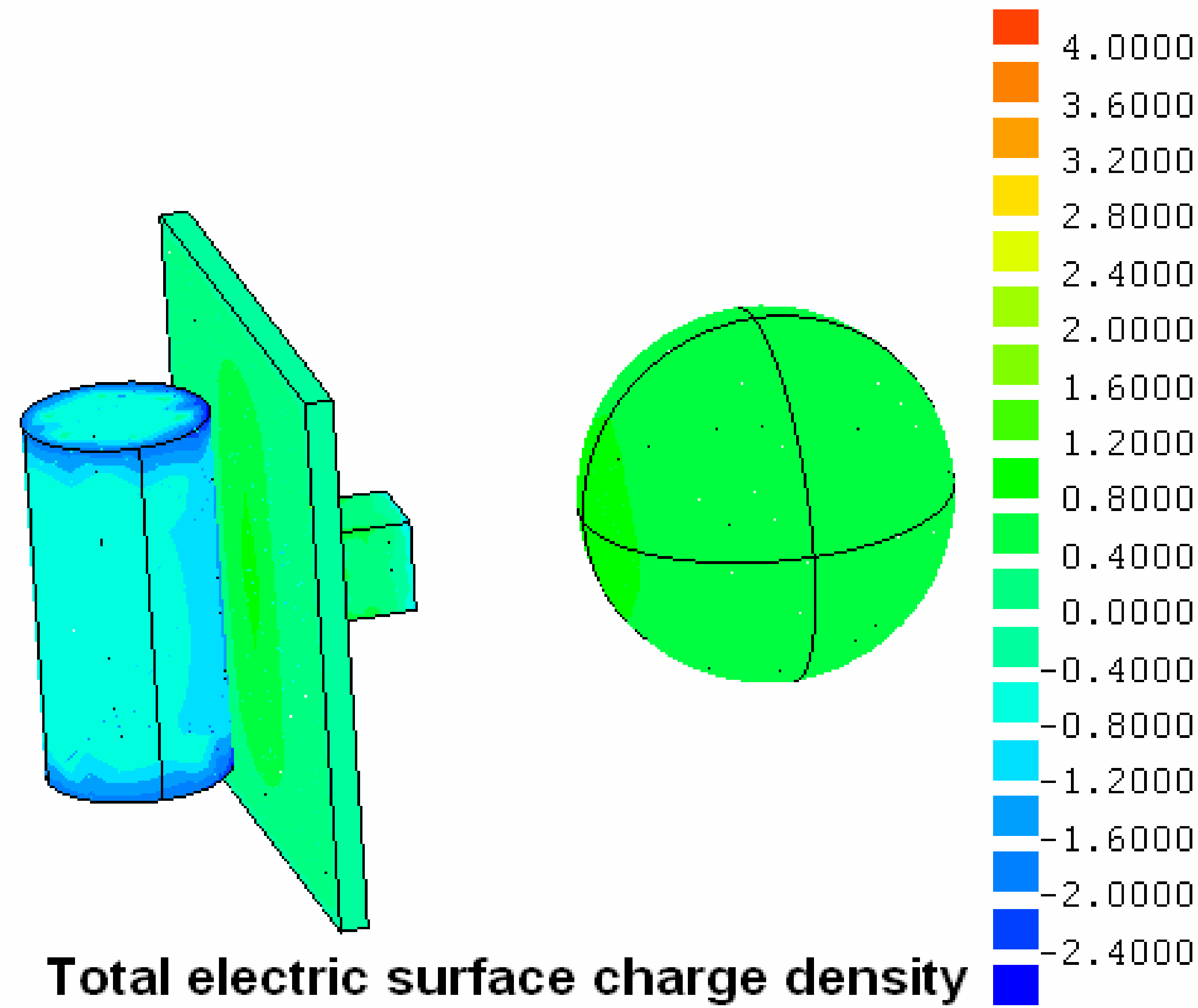
Ne=2414; Nn=1215

3D electrostatic example



Electric field (intensity and vectors)

3D electrostatic example



Total electric surface charge density

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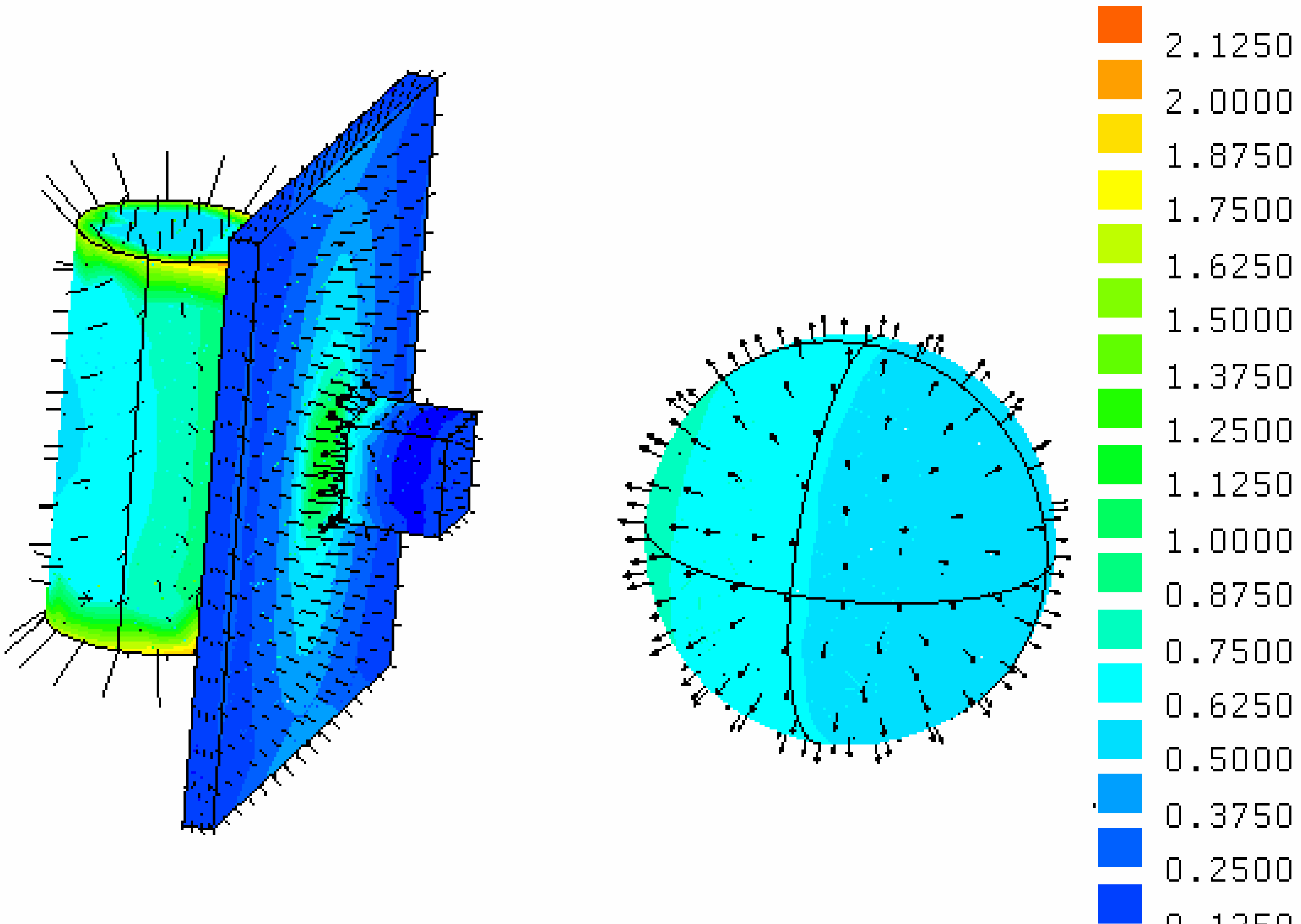
3D electrostatic example



Electric field (intensity and vectors)

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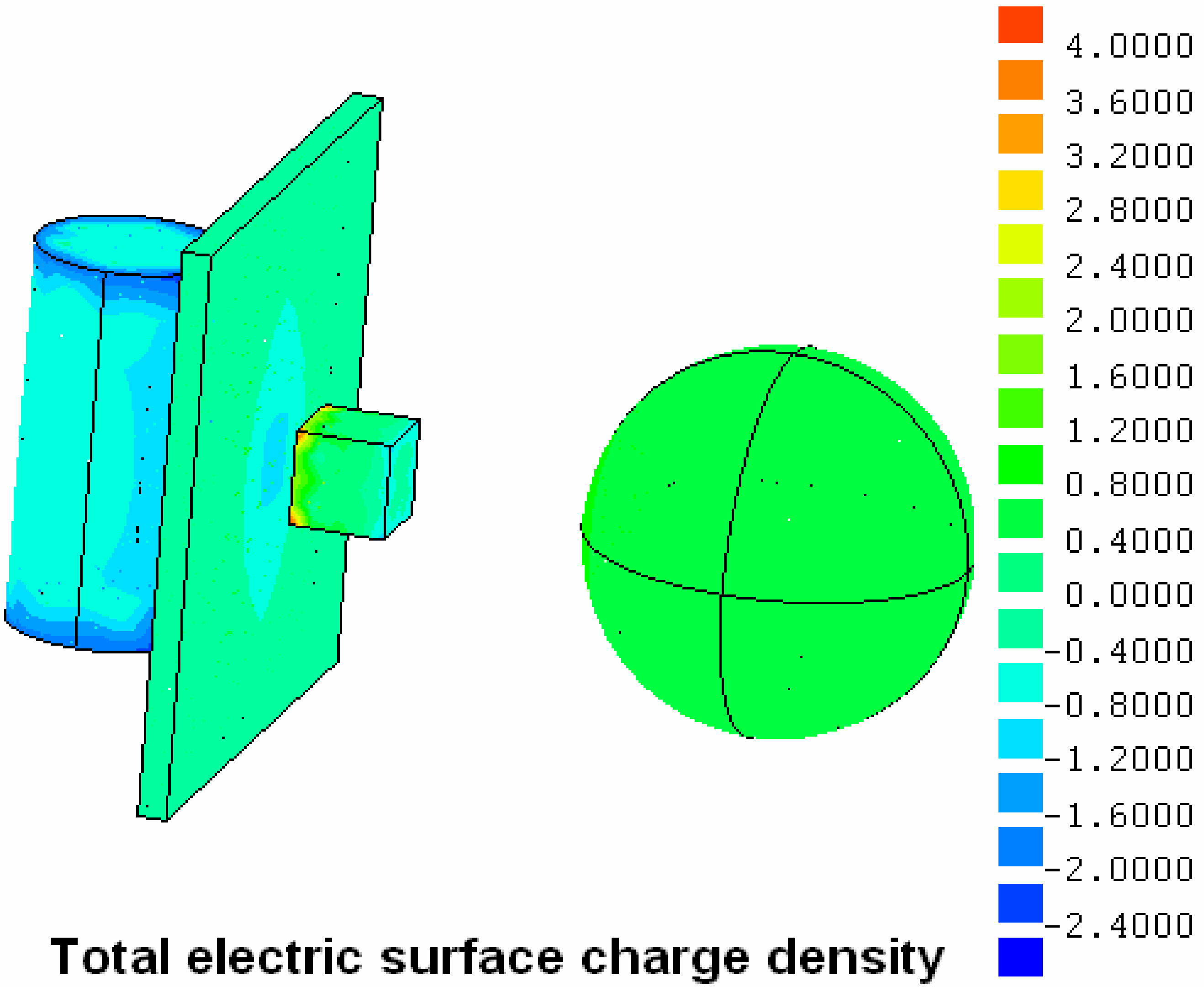
3D electrostatic example



Total electric surface charge density

3D electrostatic example

Basic solution data

Nn=1215; Ne=2414

Full matrix size = 1215 x 1215

Full matrix memory = 17MB

CPU time = 60 sec (matrix assemblage + solution + field calc.)

CPU time (matrix generation) = 32 sec

Diagonal preconditioning + GMRES (convergence in 21 iterat.)

CPU time (system solution) = 1 sec

CPU time (field calculation) = 27 sec

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Matrix compression (fast multipole technique)

Kernel expansion

$$|x-y| >> 0\,(farfield) \Rightarrow$$

$$K(x,y) \approx K_m(x,y;x_0,y_0) = \sum_{(\mu,\nu)\in I_m} K_{(\mu,\nu)}(x_0,y_0)\cdot X_\mu(x,x_0)\cdot Y_\nu(y,y_0)$$

Taylor-, Multipole-, Cebysev- expansion

$$|x-x_0| + |y-y_0| \le \eta \cdot |x_0 - y_0|$$

Farfield condition

GMRES with clustering

$$v = \tilde{A}\cdot u = N\cdot u + \sum_{(\sigma,\tau)\in F} X_\sigma^T\left(F_{\sigma\tau}\left(Y_\tau \cdot u\right)\right)$$

Matrix-vector multiplication

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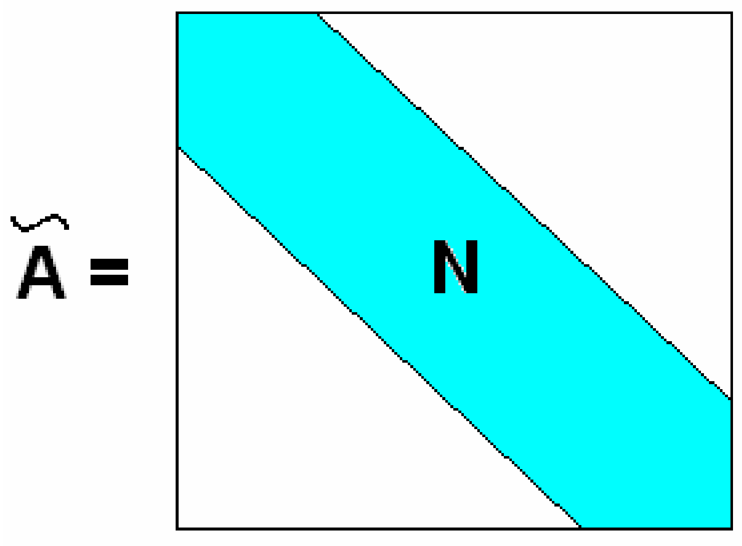
Matrix compression (fast multipole technique)



GMRES with clustering

$$v = \tilde{A} \cdot u = N \cdot u + \sum_{(\sigma,\tau)\in F} X_\sigma^T \left(F_{\sigma\tau}\left(Y_\tau \cdot u\right)\right)$$

Matrix-vector multiplication

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Matrix compression (fast multipole technique)

Basic solution data (with compression)

- Nn=1215; Ne=2414
- Full matrix size = 1215 x 1215
- Near-field matrix memory = 1MB
- CPU time = 22 sec (matrix assemblage + solution + field calc.)
- CPU time (matrix generation) = 10 sec
- Diagonal preconditioning + GMRES (convergence in 21 iterat.)
- CPU time (system solution) = 1 sec
- CPU time (field calculation) = 11 sec

Singular integrals

Point collocation

$$\iint_{(\Delta^e)} \left(K(x_i, y) \cdot N_j^e(y) \right) d\Omega, \; x_i \in \Delta^e$$

CPV integrals

$$K(x,y) \to \frac{1}{|x-y|^n}$$

$$x \to y$$

n = 1 – weak singularity

n = 2 – strong singularity

n = 3 – hyper singularity

Flat elements: analytical integration

Curved elements:

- Numerical integration (Gauss quadrature)
- Coordinate transformations (Duffy transformation)

Integral formulation of 3D eddy-currents problem

3D eddy-current problem (at industrial frequencies)

$$\nabla \times \vec{H} = \vec{J} \qquad \nabla \times \vec{E} = -j\omega \vec{B}$$ ← curl Maxwell equations

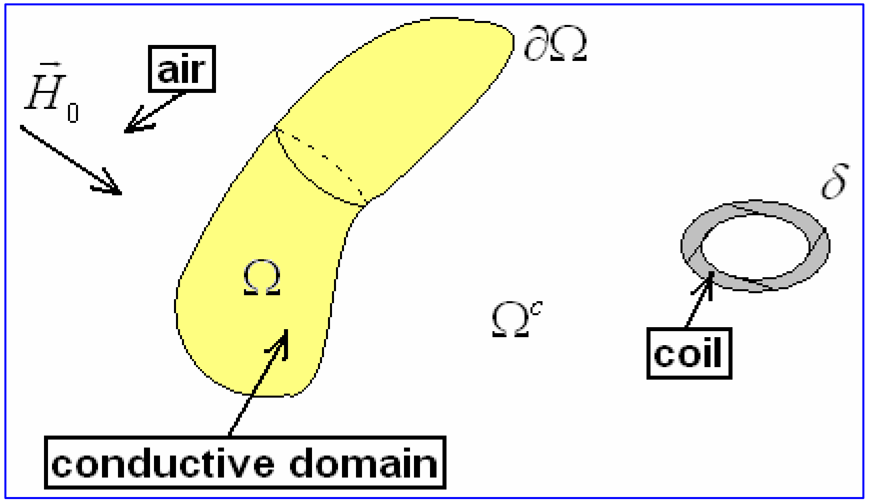
$$\vec{B} = \mu_0 \mu_r \vec{H} \qquad J = \sigma \vec{E}$$ ← Constitutive relations

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \cdot \vec{J} = 0$$ ← div Maxwell equations

$$\vec{n} \times \left(\vec{H}_1 - \vec{H}_2\right) = \vec{K} \qquad \vec{n} \times \left(\vec{E}_1 - \vec{E}_2\right) = 0$$ ← Interface conditions

Integral formulation of 3D eddy-currents problem

$\vec{H}-\varphi \quad formulation$



$\vec{H}_0$ – impressed field

$\vec{H}_\delta$ – field due to the coil

$\vec{H}_S$ – secondary mag. field

$$\vec{H}^{\Omega^c} = \vec{H}_0 + \vec{H}_\delta + \vec{H}_S$$

$$\vec{H}_S = -\nabla\varphi$$

$$\nabla\times\nabla\times\vec{H}^{\Omega} = -j\omega\mu\sigma\vec{H}^{\Omega}$$

$$\nabla\cdot\vec{H}^{\Omega} = 0 \qquad \nabla^2\cdot\varphi = 0$$

Interface conditions

$$\vec{n}\times\left(\vec{H}^{\Omega} + \nabla\varphi\right) = \vec{n}\times\left(\vec{H}_\delta + \vec{H}_0\right)$$

$$\vec{n}\cdot\left(\mu\vec{H}^{\Omega} + \mu_0\nabla\varphi\right) = \mu_0\vec{n}\cdot\left(\vec{H}_\delta + \vec{H}_0\right)$$

Integral formulation of 3D eddy-currents problem

$\vec{H} - \varphi \quad formulation$

$$-\frac{1}{2}\vec{J}(\xi) + \frac{1}{4\pi}\oiint_{(\partial\Omega)}\left[\vec{n}_\xi \times \left(\vec{J}(\eta) \times \nabla_\xi K(\eta,\xi)\right)\right] dS_\eta -$$

$$-\frac{1}{4\pi}\oiint_{(\partial\Omega)}\left[\sigma_m(\eta)\left(\vec{n}_\xi \times \nabla_\xi G(\eta,\xi)\right)\right] dS_\eta = -\left[\vec{H}_0^t(\xi) + \vec{H}_\delta^t(\xi)\right]$$

$$G(\eta,\xi) = \frac{1}{r_{\eta,\xi}}$$

$$\forall \eta, \xi \in \partial\Omega$$

$$-\frac{1}{2}\sigma_m(\xi) - \frac{1}{4\pi}\oiint_{(\partial\Omega)}\left[\sigma_m(\eta)\left(\vec{n}_\xi \cdot \nabla_\xi G(\eta,\xi)\right)\right] dS_\eta -$$

$$-\frac{\mu}{4\pi\mu_0}\oiint_{(\partial\Omega)}\left[\vec{n}_\xi \cdot \left(\vec{J}(\eta) \times \nabla_\xi K(\eta,\xi)\right)\right] dS_\eta = -\left[\vec{H}_0^n(\xi) + \vec{H}_\delta^n(\xi)\right]$$

$\vec{J}$ – virtual current

σ_m – virtual magnetic charge

$$K(\eta,\xi) = \frac{e^{-(1+j)\cdot k \cdot r_{\eta,\xi}}}{r_{\eta,\xi}}$$

$$k = \sqrt{\omega\mu_0\mu_r\sigma/2}$$

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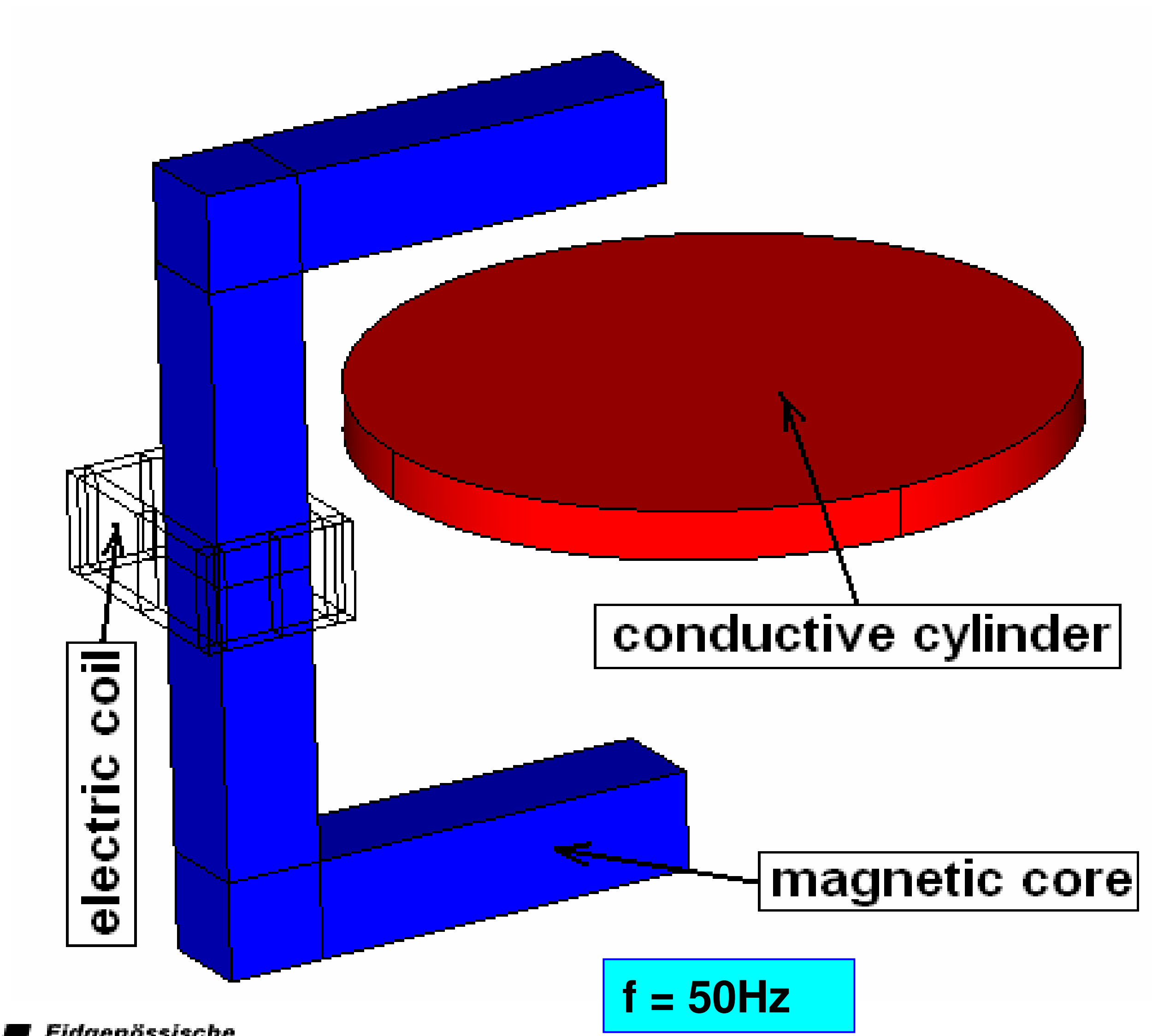
3D eddy-currents analysis example



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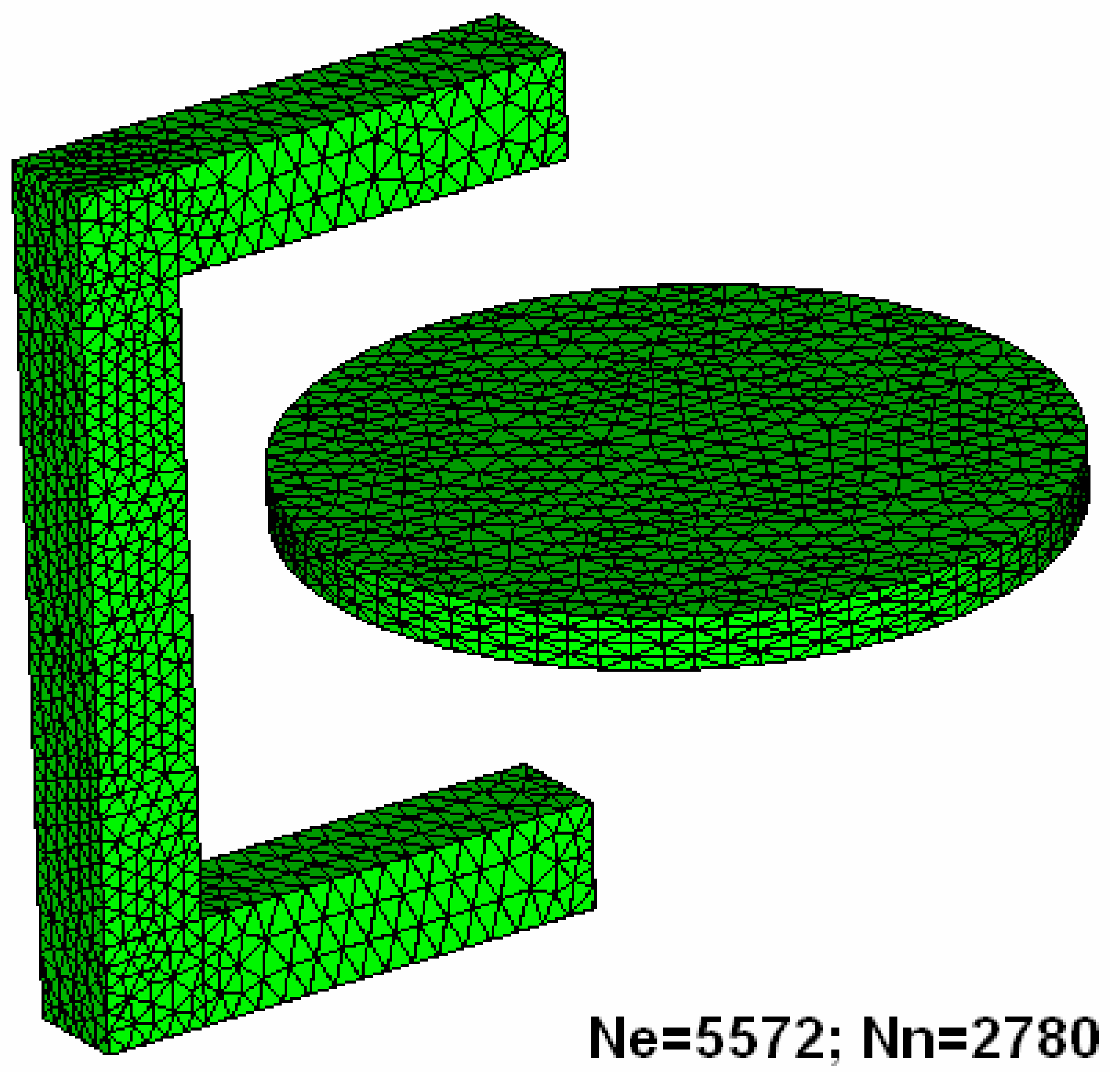

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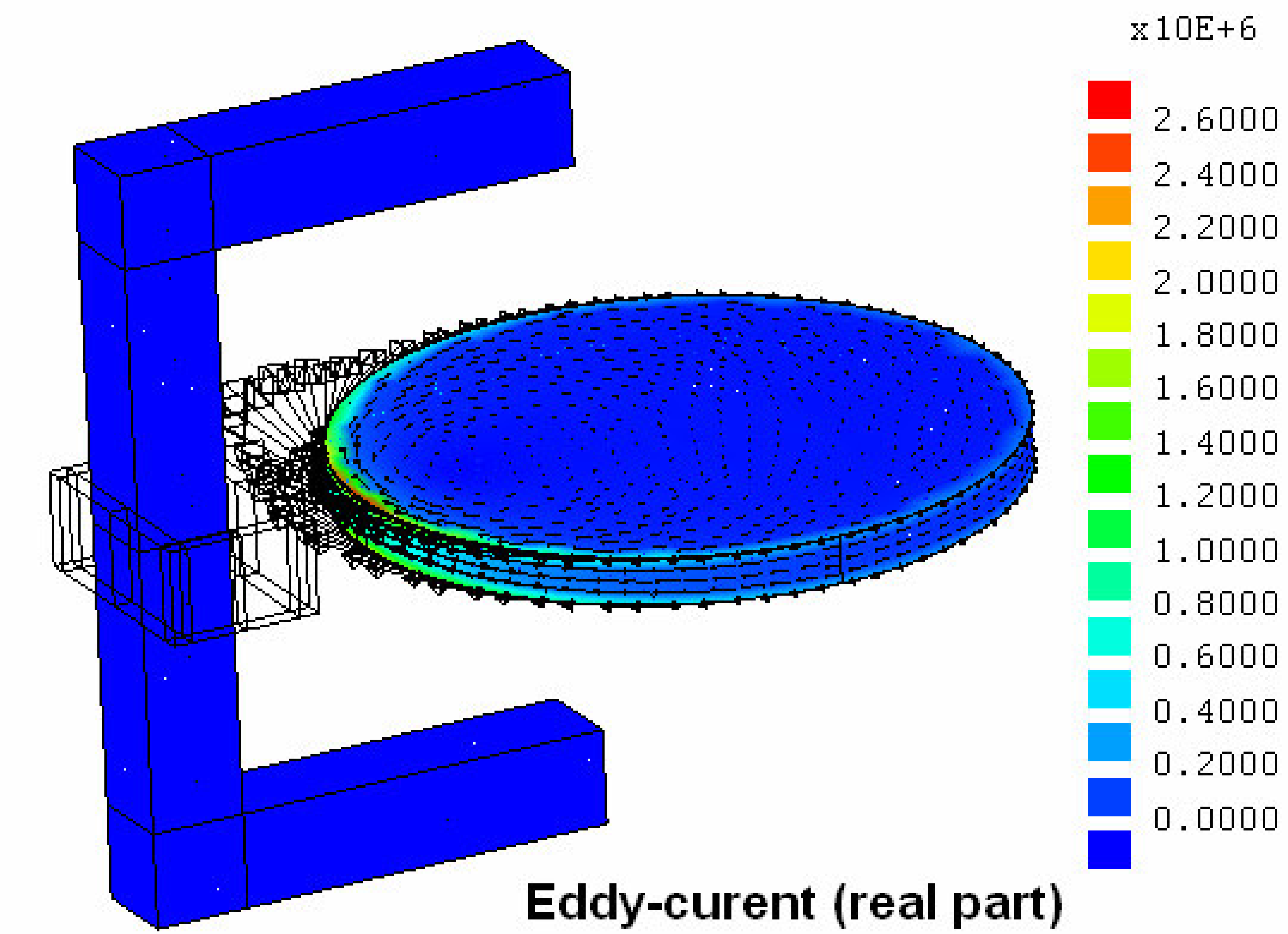

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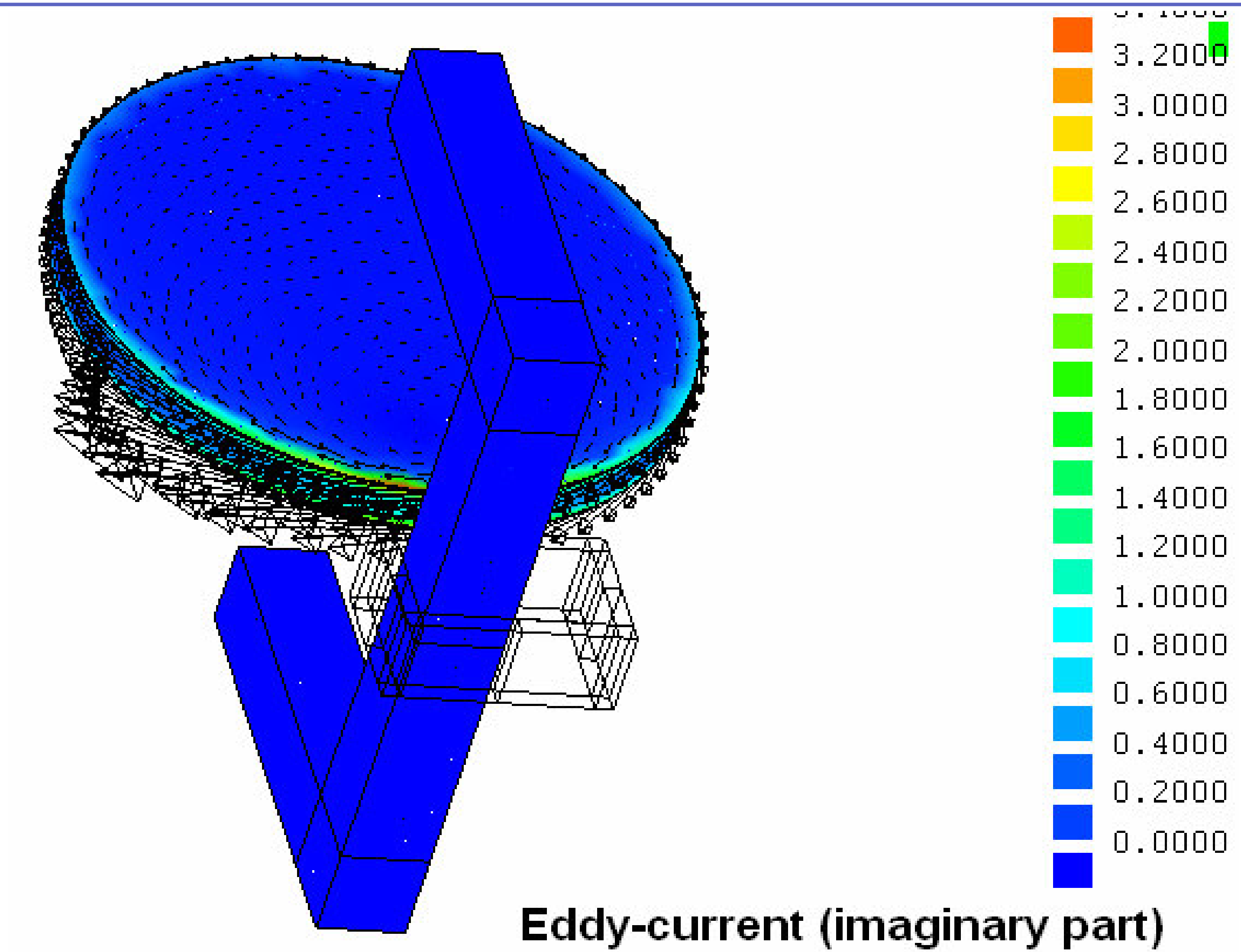

Eddy-current (imaginary part)

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3D eddy-currents analysis example

Basic solution data (with compression)

- Nn=2780; Ne=5552
- Full matrix size = 2780 x 2780
- Near-field matrix memory = 44MB
- CPU time = 100 sec (matrix assemblage + solution + field calc.)
- CPU time (matrix generation) = 47 sec
- Diagonal preconditioning + GMRES (convergence in 54 iterat.)
- CPU time (system solution) = 7 sec
- CPU time (field calculation) = 46 sec

Conclusions

- We have illustrated BEM application to 3D electrostatic and 3D eddy-currents problems
- In the case of homogeneous and linear materials BEM is dominant in comparison by FEM
- Obviously BEM is very powerful simulation tool for certain class of electromagnetic problems
- Dense matrix and singular integrals are the basic drawback of BEM
- Matrix compression techniques (fast multipole method or ACA) are promising techniques for the future of BEM.

ABB