11. Coupled Fields Analysis

11.1. Introduction

In the previous chapters we have separately analysed the electromagnetic, thermal and mechanical fields. We have discussed their sources, associated material properties and boundary conditions. At first glance these fields are independent from each other and therefore can be individually analysed. As opposed to this simplified picture, in reality the fields influence each other and this is called a coupling. Fields coupling is usually originated due to the influence of one field to the material properties or geometry valid for another field. The simulation of field coupling is not an easy task because both fields, i.e. both BVPs, have to be solved simultaneously. In this chapter the coupling between electromagnetic and mechanic fields will be analysed and an iterative method for its solution will be shown.

11.2. Fields Coupling Between Electromagnetics and Mechanics

As was already shown, the result of an electromagnetic field computation is the field distribution in the domain of our interest. Having the electromagnetic field we can compute the distribution of the electromagnetic force, i.e. its volume density acting on each elemental volume of the domain affected by the electromagnetic field. Therefore, if the computational domain is a solid body, this electromagnetic force density will act as a mechanical load on this body producing its deformation. Thus, the electromagnetic force appears to be the load for a mechanical analysis that then has to be performed in order to obtain the body deformation.

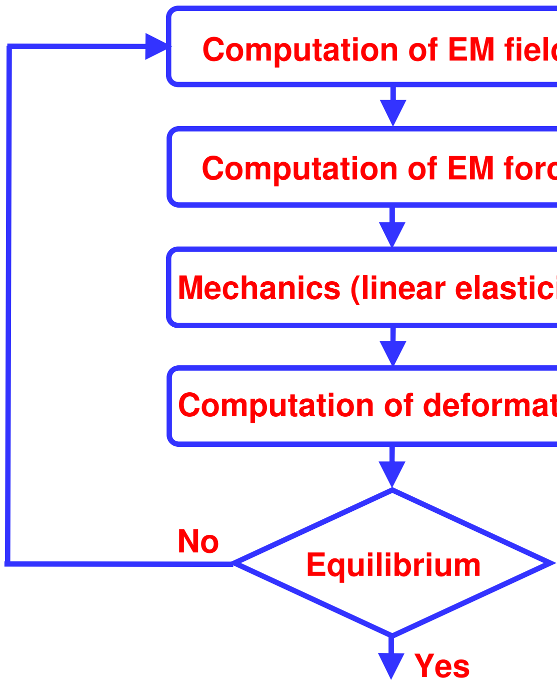


Figure 11.1. The iterative algorithm for the solution of the two-way field coupling between electromagnetics and mechanics is presented; More details can be found in the text.

As soon as we have the deformation as the result of our mechanical analysis, we can say we have solved the one-way field coupling problem. Namely, we have computed the electromagnetic field and used the results as the load for the subsequent mechanical analysis that gave us the corresponding deformation. This is already something, but is not an accurate picture of reality. Namely, the deformation of the body has an influence on the electromagnetic field as well. If we want to accurately simulate the field coupling, after the mechanical simulation has been performed, we have to go back to the electromagnetic analysis with the new (deformed) geometry of the domain. After the electromagnetic computation is finished again we have obtained the corresponding force distribution, i.e. the new load for mechanics and so on. We have actually described the iterative procedure for the solution of two-way field coupling between electromagnetics and mechanics. This algorithm is shown in Figure 11.1. As one can see, the loop of the iterative procedure is locked with the equilibrium condition. We can exit the loop only if this condition is fulfilled. The equilibrium in this algorithm is reached if the deformation of the current step is not very different from the achieved deformation in the previous step. The relative difference of the deformation[1] should usually be below a certain limit defined at the beginning of the algorithm (typical value is 10^{-4}). The parameters of the convergence of the algorithm will be discussed later in details.

11.3. Coupled-Field Analysis in Shunt Capacitive MEMS Switch

In Section 8.4, the shunt capacitive MEMS switch [1] already was described in detail. Its geometry with the corresponding dimensions is shown in Figure 8.2. As was already explained, the membrane of the switch is pulled in with the electrostatic force produced by the electrostatic field in the air-gap between the membrane and the middle conductor of the coplanar waveguide (CPW) [1, 2, 3]. For a certain DC voltage applied to the structure, the electrostatic force will deform the membrane. The elasticity of the membrane will act against the electrostatic force and eventually the membrane will deform, taking the position of equilibrium. Thus, the coupling of the electrostatic and mechanical fields has to be resolved.

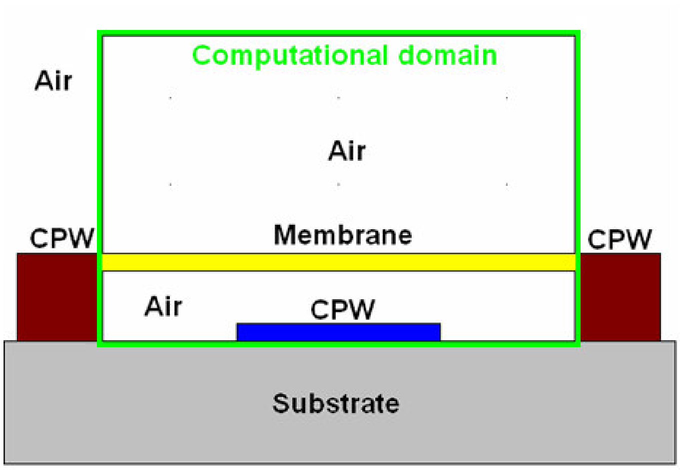
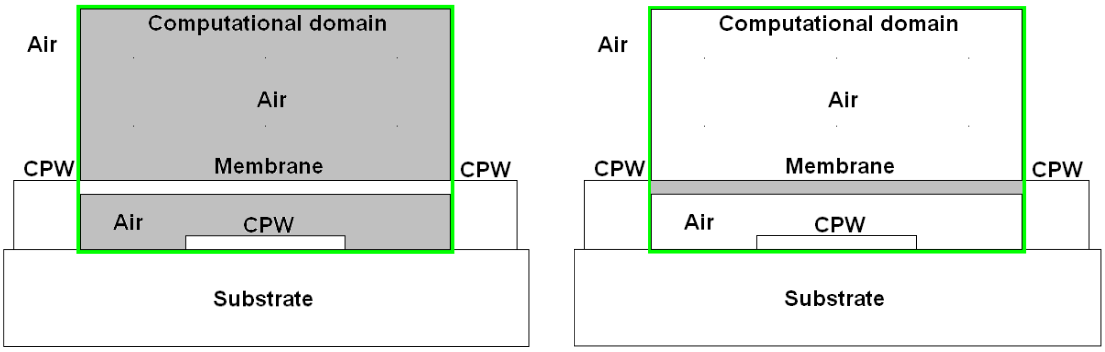


Figure 11.2. The cross-section of the shunt capacitive MEMS switch is presented; The computational domain (green box) is reduced compared to the domain of the mechanical analysis presented in Chapter 8 (more details can be found in text); The abbreviation CPW means the coplanar waveguide (the CPW system consists of two grounded conductors (brown) on the sides and one signal carrying conductor (blue) in the middle [2, 3]).

We have already presented the numerical computation of the electrostatic and mechanical fields. Therefore, we will focus here only to the aspects of field coupling. Similar to the mechanical analysis of the MEMS switch presented in Chapter 8, we will limit our analysis to the 2D model, i.e. the analysis of the cross-section presented in Figure 11.2. Since the length of the membrane in the direction perpendicular to the plane of the cross-section is much larger than the thickness of the membrane and the air gap taken together, this 2D

[1] The relative difference of the deformation is the norm of the deformation vectors difference of the consecutive step divided by the norm of the obtained deformation vector in the current step.

approximation is rather close to reality. As one can see in Figure 11.2, the computational domain represented by the green box is rather reduced compared to the computational domain of the same device used in the mechanical analysis presented in Chapter 8 (the computational domain there is presented in Figure 8.3). The reason is very simple and can be seen in Figure 8.4 showing the results of the mechanical analysis. Namely, the conductors of the CPW (brown coloured in Figure 11.2) are much larger than the thickness of the membrane (yellow coloured in Figure 11.2). Thus their deformation due to the electrostatic force is negligible compared to the deformation of the membrane. On the other hand, from the electrostatic field viewpoint, they are at zero potential (grounded) and their influence on the electrostatic field in the air gap under the membrane is also negligible. Therefore, they can be omitted from our analysis.



(a) (b)

Figure 11.3. The computational domain (gray area) of the electrostatic (a) and mechanical analysis (b); In the electrostatic analysis the membrane is set to the potential zero and the middle conductor of the CPW is set to the input DC voltage (V_{IN}); In the mechanical analysis only membrane is under consideration and the touching surfaces between the membrane and the conductors of the CPW are set to be fixed (a mechanical constraint or zero displacement BC).

The substrate is also omitted because it has no meaning in either the mechanical (only membrane is analyzed) or electrostatic analysis (actually it has a small influence on the electrostatic field in the gap due to its dielectric constant $\varepsilon_r \approx 10$, but in order to keep the problem size and CPU time small it has been removed from the model). From a mechanical point of view only the membrane is subjected to elastic deformation and therefore its geometry defines the computational domain. The situation is completely different in the case of electrostatics. Namely, the metal parts are excluded from the model (the electric field in a perfect conductor is equal to zero) and the air gap is included in the model. Although, from an electrostatic point of view, the air above the membrane has no influence, it has been taken into the computational domain for the reason of mesh movement and deformation that will be explained later. Thus we have defined the computational domains for both the mechanical and electrostatic analyses and they are shown in Figure 11.3.
The next step of our analysis will be to generate the mesh. Since our structure has large disproportion between its width (520 μm) and height (5 μm), i.e. it has a large aspect ratio the process of mesh generation is not an easy task. The best numerical results for such structures can be obtained by using a regular mesh, i.e. by using the regular rectangular elements. Such a mesh was generated for our geometry and is shown in Figure 11.4. We have to be very careful when we define the divisions of a regular grid. It is important to keep in mind that our

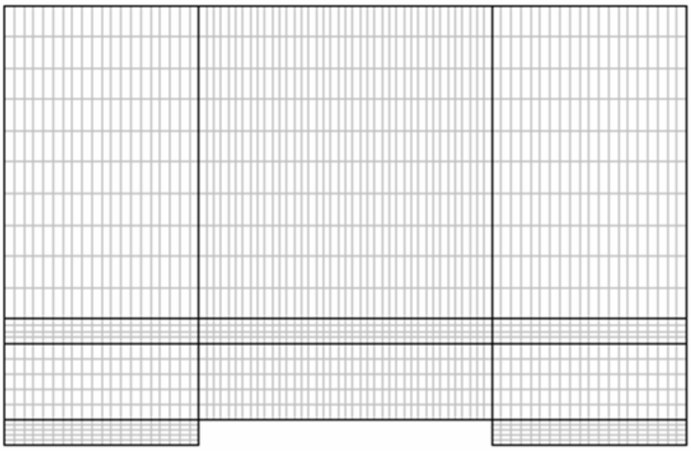


Figure 11.4. The regular mesh covering our computational domain is shown; Such a mesh can partially compensate the negative numerical effect of the large geometric aspect ratio of our structure.

structure size is not proportional at all in the x- and y-directions. Otherwise, too large of a division in the y-direction will contribute to our initially very bad aspect ratio. The rule is very simple. It is necessary to put as many elements as possible in the x-direction keeping the division in the y-direction as small as possible.

After we have defined the mesh we have to think about the coupled-field simulation, i.e. we have to numerically implement the algorithm presented in Figure 11.1. Having the mesh given in Figure 11.4 and separate mechanical and electrostatic field solvers, it is possible to go once through our algorithm from Figure 11.1. The mechanical analysis will have the deformation of our membrane as an output. Theoretically speaking, we could then regenerate the mesh on the deformed structure which is very difficult if we speak about a regular rectangular grid. Therefore, we need a slightly different algorithm if we want to keep using

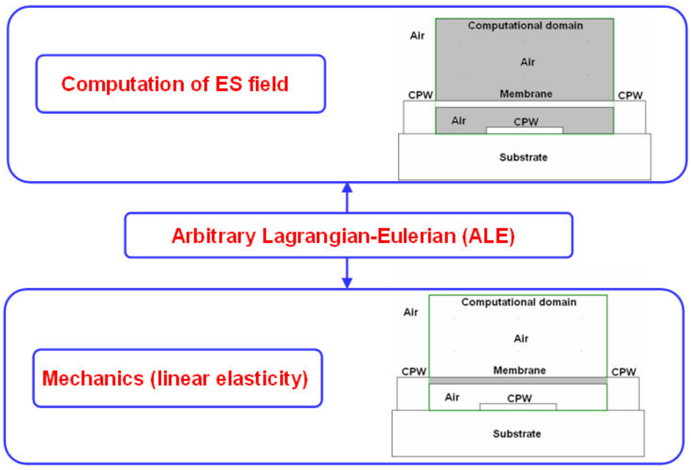


Figure 11.5. The role of the Arbitrary Lagrangeian-Eulerian (ALE) method in the field coupling is presented; The ALE method allows us to consider our mesh as movable and/or deformable; Thus our coupled analysis becomes a single but nonlinear problem.

our regular mesh. As a possible solution to this problem a very logical approach can be suggested. Namely, instead of regenerating the mesh, it is possible to keep the original regular mesh. Obviously this original mesh cannot be any more geometrically static and we have to be able to consider the mesh as *movable* and/or *deformable*. In order to do so, we have to apply the so-called *Arbitrary Lagrangeian-Eulerian (ALE)* method [4]. The theoretical background of this method is beyond the content and purpose of this script and can be found in the given reference [4]. Using the ALE method we can avoid the algorithm presented in Figure 11.1 and avoid the mesh regeneration after each mechanical analysis. The ALE interface can couple the electrostatic and mechanical analysis and, using a nonlinear solver, obtain the solution of this complete coupled problem. Such a procedure is shown in Figure 11.5.

Using the algorithm from Figure 11.5, the parametric study of our coupled problem is performed with respect to the input DC voltage U_{IN} as a parameter. For various different values of the input voltage, the coupled electrostatic and mechanical fields have been computed and the equilibrium position of the membrane has been determined. Some characteristic results are presented in Figure 11.6. This analysis can be used to determine the so-called pull-in voltage, i.e. the DC voltage at which the membrane touches the middle conductor (the on state of the switch).

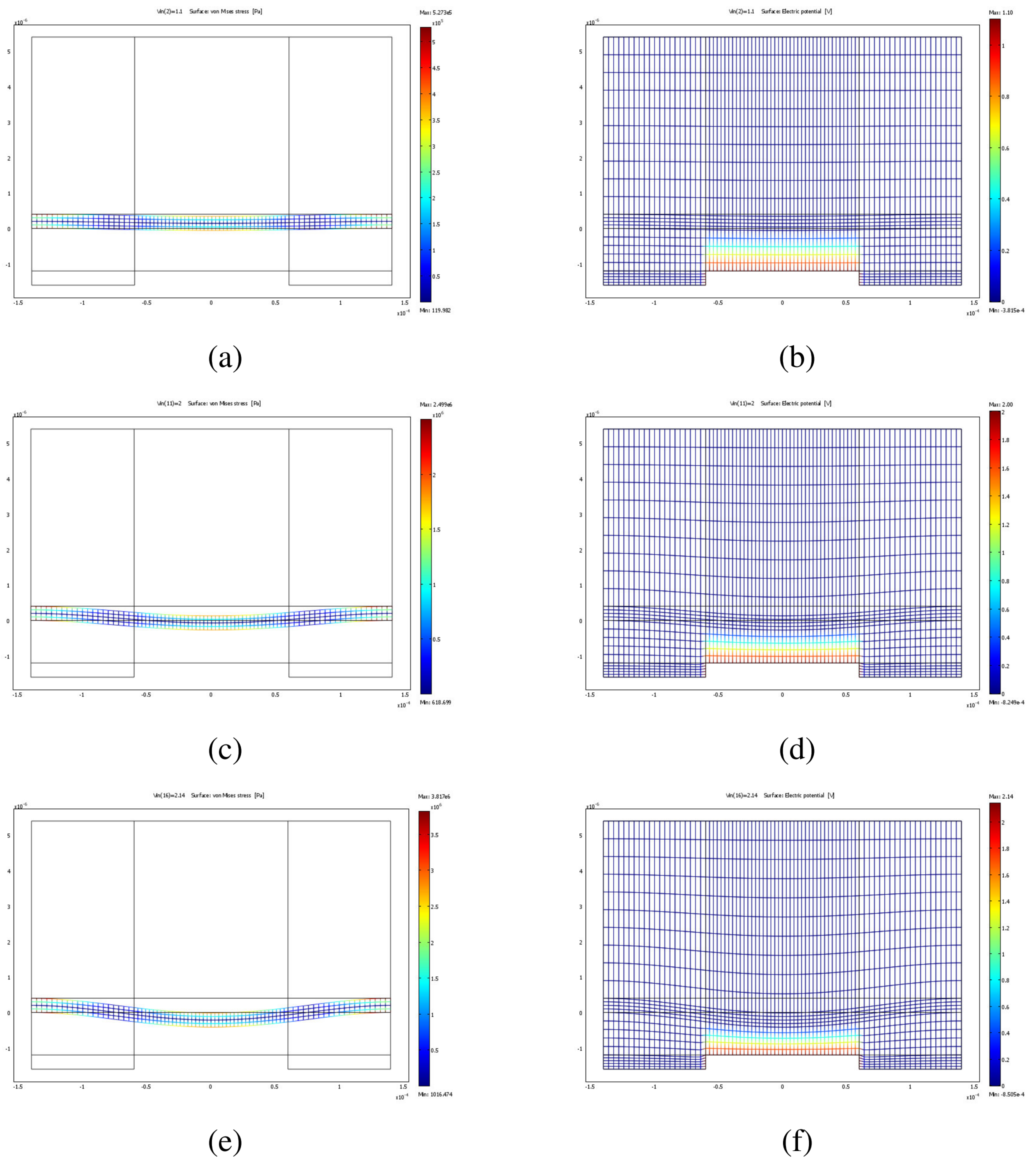


Figure 11.6. The results of the coupled field analysis using the ALE method and a nonlinear solver are presented; The parameter U_{IN} had the following values: 1.1V (a, b), 2.0V (c, d) and 2.14V (e, f); The stress over the membrane (a, c, e) together with the potential over the entire computational domain (b, d, f) are presented; The deformation of the mesh is visible.

Since the ALE method cannot be used to really simulate the situation when the membrane touches the middle conductor, because the elements below the membrane will then collapse, this method can only be used to estimate the pull-in voltage using the graph presented in Figure 11.7.

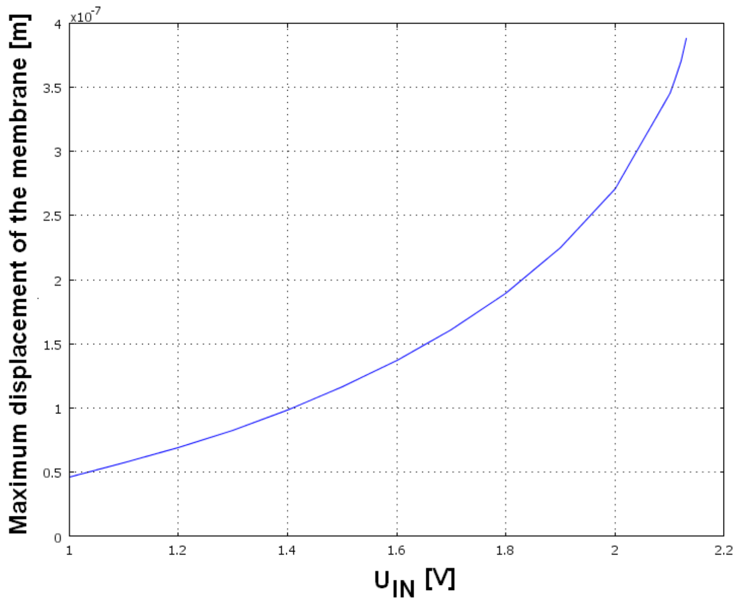


Figure 11.7. The result of the parameter study of the coupled-field problem using the ALE method; The maximum deformation of the membrane with respect to the input DC voltage is presented; The extrapolation of this curve can help us to estimate the pull-in voltage.

11.4. Parameters of Convergence

The solution of the complete nonlinear coupled-field problem using the ALE method has very sensitive convergence. Among many others the following parameters have the most significant influence on the convergence:

- quality of the mesh (aspect ratio of the elements),
- material properties,
- severity of the deformation (the level of the nonlinearity),
- step size in a parameter study.

The first two parameters are already explained in previous chapters. The severity of the deformation can produce a failure of the nonlinear solver due to a large discrepancy between the initial solution (zero displacement) and equilibrium. To overcome this problem it is better to start with a modest deformation (small value of the input voltage) and then step by step increase it to try to reach the desired value. In close connection with it is the step size of the parameter study which should be, for the same reason, significantly small. If the material configuration, mesh quality and severity of the deformation produce a highly nonlinear problem, a careful selection of the parameters of the nonlinear solver (especially the relaxation factor and adaptability) can be helpful as well. However, convergence can sometimes be so difficult and sensitive that only a patient fine tuning of the parameters of the nonlinear solver and mesh quality can resolve the problem.

11.5. Concluding Remarks

We have presented the general idea of field coupling along with a simple solution algorithm. The presented methodology has been used to solve the field coupling problem of the shunt capacitive MEMS switch. The coupling between the electrostatic and mechanical fields has been analyzed. In addition, the role of the ALE procedure for a movable and/or deformable mesh has been explained. The obtained results confirmed the efficiency of the presented algorithms. Apparently, even at this simple level used for educational purposes, the presented solution algorithm for field coupling can be successfully used for the estimation of the pull-in voltage of the shunt capacitive MEMS switch.

Although our analysis has been focused on the coupling between the electromagnetic and mechanical fields, it is worth mentioning that various other ways of coupling occur in real-life devices such as the coupling between electromagnetic and thermal fields, thermal field and fluid dynamics and so on. Apparently, a highly accurate prediction of the device behaviour is not possible without taking into account these coupling effects. Thus, field coupling appears to be the most recent challenge for modern simulation technology as well as for the available hardware.

11.6. References

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