

Exercise 11:

Determine the capacitance per unit-length of a parallel plate capacitor filled by a dielectric material with ϵ_r . The plates are separated by a distance d and the width of the plate is L .

- Solve the problem analytically. (By neglecting fringing fields.)
- Solve the problem by using a FD scheme. (By neglecting fringing fields.)
- Solve the problem by using COMSOL. (By allowing fringing fields.)
- Compare the results obtained from COMSOL with the analytical result as a function of $\epsilon_r = \text{linspace}(1, 100, 20)$.

Solution

a) In order to determine the capacitance of a structure, the steps below must be followed:

- Choose the coordinate system (cartesian, cylindrical or spherical?)
- Put charges $+Q$ and $-Q$ on the conductors.
- Find the electric field by Gauss' Law.
- Find the potential difference between the conductors by:

$$V_{12} = - \int_1^2 \mathbf{E} \cdot d\mathbf{l} \quad (1)$$

5) Use the following to determine the capacitance:

$$C = \frac{Q}{V_{12}} \quad (2)$$

For the specific example, have a look at Fig.

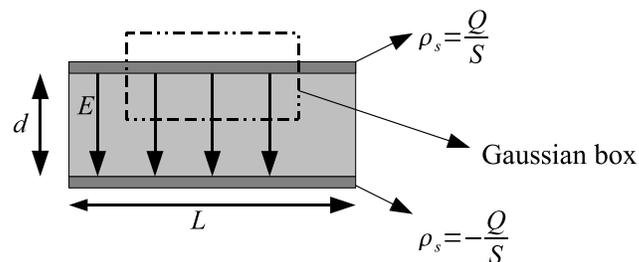


Figure 1: The schematic of the parallel plate capacitor with the direction of the E-field and the location of the Gaussian surface. S is the total surface of one of the conductors.

By using Gauss' Law, the magnitude of the E-field between the conductors is found by:

$$\mathbf{E} = \frac{Q}{\epsilon_r \epsilon_0 S} \quad (3)$$

Therefore the potential difference between the two conductors is found by:

$$V_{12} = - \int_1^2 \frac{Q}{\epsilon_r \epsilon_0 S} \cdot dl = \frac{Qd}{\epsilon_r \epsilon_0 S} \quad (4)$$

Finally the capacitance is found by:

$$C = \frac{Q}{V_{12}} = \frac{\epsilon_r \epsilon_0 S}{d} \quad (5)$$

b) The equation to be solved is Laplace's Equation. Since there are no fringing fields allowed, the equation can be simplified to 1D. Therefore the equation to be solved by FD becomes:

$$\nabla^2 V = 0 = \frac{d^2 V}{dx^2} \quad (6)$$

By applying a forward-backward FD scheme, we obtain the following numerical differentiation rule:

$$0 = \frac{V(x+h) - 2V(x) + V(x-h)}{h^2} \quad (7)$$

which gives the following update scheme:

$$V(x) = \frac{V(x+h) + V(x-h)}{2} \quad (8)$$

where h is the width of discretization. For an application of this scheme in Matlab, please refer to the file *cap_FD.m* in the package.

c) Please refer to `Capacitor_same_dielectric_linear_42.mph` for a solution in Comsol.

d) Please refer to `Capacitor_same_dielectric_linear_42.mph` for a solution in Comsol. Export the solution (the integration of the surface charge density over one of the faces of the capacitor) to Matlab, then run

```
[analytic_c,numerical_c]=comparison_caps(0.2,1.0,kk);
```

for a comparison of the results obtained from Comsol and the analytical calculations. (Here the result is stored in the variable called `kk`.)

Exercise 12:

Determine the capacitance per unit length between two long, parallel, circular metallic wires. The radii of the wires are the same and equal to r and the separation between the centers of the wires is equal to d .

- Solve the problem analytically.
- Use the results obtained in part (a) to solve the problem by OpenMaX.
- Solve the problem by using COMSOL.

Solution

a) By using the results from last week, the location of the point sources are found to be:

$$d - d_i = 0.5(d + \sqrt{d^2 - 4r^2}) \quad (9)$$

where d_i is the distance between the point charge and the center of the cylinders. Then, the capacitance of the structure is found to be:

$$C = \frac{\pi\epsilon_0}{\cosh^{-1}\left(\frac{d}{2r}\right)} \quad (10)$$

b) Please refer to the files `Wires000.PRO` and `Wiresmmp_dir.PRO` for the solutions generated by OpenMAX, with the optimized location of the point charges (solution from part a) is used) and automatically distributed point charges, respectively.

c) Please refer to the file `cylinders.mph` for the Comsol result.