Exercise[1] 7

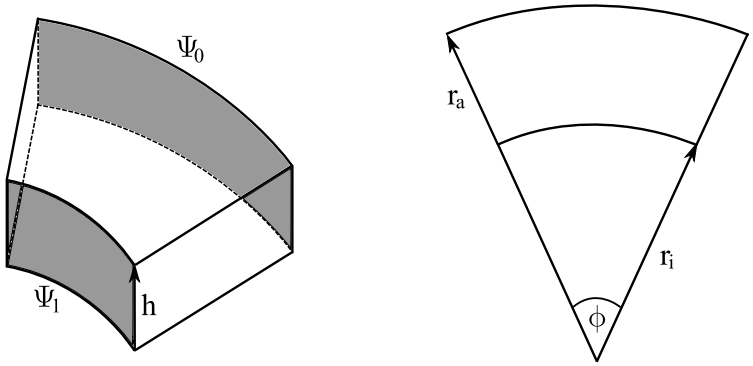

Figure 1: sketch of cylindrical sector resistor.

A three dimensional resistor of homogenous conductivity $\sigma = 1e7\ \frac{\mathrm{S}}{\mathrm{m}}$ is clamped between two electrodes with a potential difference of $\Psi_1 - \Psi_0 = 10\mathrm{V}$ as depicted in figure 1 with the following parameter:

$$\phi = \frac{\pi}{4}, \qquad r_a = 5\ \mathrm{cm}, \qquad \mathrm{r_i} = 2\ \mathrm{cm}, \qquad \mathrm{h} = 1\ \mathrm{cm}.$$

For the following calculations neglect the fringing fields of the setup and assume a time invariant (static) electric fields.

a) Derive the governing partial differential equation from the Maxwell equation!
hint: $\nabla \cdot \left(\nabla \times \vec{A}\right) = 0, \qquad \nabla \times (\nabla \alpha) = 0$

b) What are the boundary conditions for each of the six areas?

c) Determine analytically the field potential $\Psi(\rho, \phi, z)$ within the resistor. Utilise the symmetric structure of the problem for simplified calculations.
hint: $\nabla^2 \Psi = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial \Psi}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 \Psi}{\partial \varphi^2} + \frac{\partial^2 \Psi}{\partial z^2}$

d) Compute the current through the resistor!

e) Compute overall resistance R of the resistor!

f) Solve the problem by Comsol. Compare the solution of Comsol with the analytical solution you obtained in part c! (This part was not asked in the exam)

[1] exam problem fall 2008