## Exercise 13:

A system of coupled differential equations is given by

$$\begin{aligned} \frac{\partial u}{\partial x} &= \frac{\partial f}{\partial t} \\ \frac{\partial f}{\partial x} &= u \end{aligned}$$

Answer the following questions:

a) Decouple the system of differential equations.

b) Find a FD update scheme for the decoupled differential equation for $f$ using central differences for derivatives towards $x$ and forward differences for derivatives towards $t$.

c) Find a FD update scheme for the coupled differential equations. Use central differences for derivatives towards $x$ and forward differences for derivatives towards $t$.

d) Under which condition are schemes b) and c) equivalent?

e) Show that the accuracy of the used forward derivatives is $O(\Delta t)$.

f) Improve scheme b) such that it shows $O(\Delta t^2)$ and $O(\Delta x^2)$ accuracy.

## Exercise 14:

Given is Poisson's equation $\frac{d^2\Phi}{d^2x} + \frac{d^2\Phi}{d^2y} = f(x, y)$

a) What is the difference between Laplace's equation and Poisson's equation?

b) Give two examples where the Poisson equation is used to describe physics and indicate the meaning of the involved fields.

c) Write the Dirichlet and Neuman boundary conditions for the potential $\Phi$.

d) What physical conditions are given by Dirichlet and Neuman boundaries in the two examples of b)? (Give for each physics a practical example of a Dirichlet and a Neuman boundary condition!)

e) Approximate the Laplace's equation with the FD method! The mesh distances are $h_x$ and $h_y$.

f) Explain the staircasing effect in FD?