Method of Auxiliary Sources for Optical Nano Structures

K.Tavzarashvili(1), G.Ghvedashvili(1), D. Kakulia(1), D.Karkashadze(1,2),
(1)Laboratory of Applied Electrodynamics, Tbilisi State University, Georgia
(2)EMCoS, EM Consulting and Software, Ltd, Tbilisi, Georgia,
e-mail: david.karkashadze@emcos.ge
The Content and Purpose

- Conventional interpretation of MAS applied to the solution of electromagnetic scattering and propagation problems.
- The general recommendations for the solution of these problems.
- An application of the method to specific problems for the single body and a set of bodies of various material filling.
- The application areas of MAS, its advantages and benefits
Mathematical Background of the Method of Auxiliary Sources (MAS)

The name “MAS”, currently used, did not appear at once. The authors themselves adhered to the names:

- “The Method of Generalized Fourier Series” [1-5]
- “The Method of Expansion by Fundamental Solutions [7,8]


“A common idea of these works is a basic theorem of completeness in $L_2(S)$ … of infinite set of particular solutions, generated by the chosen fundamental or other singular solutions…” [7] of wave equation (D.K.)
Electromagnetic Scattering and Propagation as the Boundary Problem

The main goal of problem is to find vectors of secondary electromagnetic field \( \{\overline{E}_m, \overline{B}_m, \overline{D}_m, \overline{H}_m\}_{m=0}^M \) in each bounded, simply connected domains \( \{\Gamma_m\}_{m=0}^M \), confining with the set of smooth, closed surfaces \( \{S_{m,n}^M\}_{m,n=0} \) (interfaces between neighbouring \( m \) and \( n \) domains), while the primary field \( \{\overline{E}^0, \overline{H}^0, \overline{D}^0, \overline{B}^0\} \) is given.

In corresponding domain secondary or primary electromagnetic field should satisfy:

a) Maxwell’s equation;

b) some type of constitutive relation among field vectors – \( \overline{D}_m = \mathcal{F}(\overline{E}_m, \overline{B}_m), \overline{H}_m = \Phi(\overline{E}_m, \overline{B}_m) \);

c) boundary conditions \( \hat{L}_m^n(\overline{E}_m, \overline{H}_m, \overline{E}_n, \overline{H}_n) = 0 \) on \( \{S_{m,n}^M\}_{m,n=0} \) surfaces;

d) in unbounded free space \( \Gamma_0 \) field vectors should satisfy the radiation condition.
Method of Auxiliary Sources (MAS) (simple 2D case)

\[ \Delta E_{\text{scat}}^{\text{scat}}(x, y) + k^2 E_{\text{scat}}^{\text{inc}}(x, y) = 0, \quad P(x, y) \in \Gamma_0 \]

1) Everywhere dense points on the surface \( S^0 \) - \( \{ r_n^0 \}_{n=1}^{\infty}, \quad r_n^0 \in S^0 \)

2) Fundamental solutions of Helmholtz equation:

\[
\Delta \varphi_n(\vec{r} - \vec{r}_n^0) + k^2 \varphi_n(\vec{r} - \vec{r}_n^0) = -\delta(\vec{r} - \vec{r}_n^0) \\
\varphi_n(kR^0_n) = \frac{i}{4} H^{(1)}_0(kR^0_n), \quad R_n^0 = \sqrt{(x-x_n^0)^2 + (y-y_n^0)^2}
\]

3) Construction of the set of fundamental solutions of wave equation with radiation centers on surface \( S^0 \):

\[
\{ \varphi_n(r_s, r_n^0) \}_{n=1}^{\infty} \Rightarrow \{ \psi_n(r_s, r_n^0) \}_{n=1}^{\infty}
\]

**Theorem:** It can be shown [*], that for an arbitrary smooth surface \( S \) (in the Lyapunov sense) one can always find the auxiliary surface \( S^0 \) such that the constructed set of functions is complete and linearly independent on \( S \) in the functional space \( L_2 \).

Method of Auxiliary Sources (MAS) (simple 2D case)

Any continuous function on $S$ can be expanded in terms of the first $N$ functions of the given set of fundamental solutions:

$$E_z^{\text{scat}}(x_s, y_s) = \sum_{n=1}^{N} a_n H_0^{(1)} \left(k\sqrt{(x_s - x_n^0)^2 + (y_s - y_n^0)^2}\right)$$

Properties of this set of fundamental solutions guarantee existence of corresponding coefficients $\{a_n\}_{n=1}^{N}$ providing the best in $L_2(S)$ mean-square approximation of any continuous function on $S$:

Corresponding discrepancy:

$$\left( \int_{S} \left| E_z^{\text{inc}}(x_s, y_s) - \sum_{n=1}^{N(\varepsilon)} a_n H_0^{(1)} \left(k\sqrt{(x_s - x_n^0)^2 + (y_s - y_n^0)^2}\right) \right|^2 dS \right)^{\frac{1}{2}} < \mathcal{E};$$

when $N(\varepsilon) \to \infty$, then $\varepsilon \to 0$
Computational Procedures for Determination MAS Unknown Coefficients

- **Gram-Schmidt Orthogonalization** approach;
- **MAS-MoM-Galerkin** approach;
- Method of **Colocation**;
- .....
Gram-Schmidt Orthogonalization Approach

\[ \left\{ \mathbf{G}_m(R_m) \right\}_{m=1}^{M} \Rightarrow \left\{ \overline{\psi}_m(r_s) \right\}_{m=1}^{M}, \quad L \left( \overline{\psi}_m(R_m) \right) = I \cdot \sum_{n=1}^{m} \delta(R_n) \]

From Fourier Theorem - The best expansion (in \( L_2 \)) of given vector function \( H(x_s,y_s,z_s) \) on surface \( S \):

\[ H(x_s,y_s,z_s) = \sum_{m=1}^{M} \overline{\psi}_m(s) \cdot b_m \]

\[ \sqrt{\int_{S} \left| H(s) - \sum_{m=1}^{M} \overline{\psi}_m(s) \cdot b_m \right|^2 ds} = \min(L_2) \]

Scalar product definition:

\[ \int_{S} (\overline{\psi}_m \cdot \overline{\psi}_k) ds \iff (\overline{\psi}_m \cdot \overline{\psi}_k) = \begin{cases} 0, & m \neq k \\ \overline{F}_m, & m = k \end{cases} \]

Orthogonalization procedure

\[ \overline{\psi}_1(s) = \mathbf{G}_1(s), \]
\[ \overline{\psi}_2(s) = \mathbf{G}_2(s) - \overline{\psi}_1(s) \cdot \overline{A}_{21}, \]
\[ \overline{\psi}_3(s) = \mathbf{G}_3(s) - \overline{\psi}_1(s) \cdot \overline{A}_{31} - \overline{\psi}_2(s) \cdot \overline{A}_{32}, \]
\[ \ldots \]
\[ \overline{\psi}_m(s) = \mathbf{G}_m(s) - \sum_{k=1}^{m-1} \overline{\psi}_k(s) \cdot \overline{A}_{mk}; \]
\[ \ldots \]

\[ \overline{A}_{mn} = \overline{F}_n^{-1} \left( \overline{C}_{nm} - \sum_{k=1}^{n-1} \overline{A}_{nk}^T \cdot \overline{A}_{mk} \right), \quad (\overline{A}_{mn} = 0, m \leq n); \]
\[ \int_{S} (\overline{G}_m \cdot \overline{G}_k) ds = \overline{C}_{mk}, \int_{S} (\overline{F}_k \cdot \overline{F}_k) ds = \overline{F}_k; \]

\[ b_q = \overline{F}_q^{-1} \cdot (\overline{\psi}_q \cdot H) \]

MAS-MoM-Galerkin Approach

MoM-like representation of current on the auxiliary surface

\[
\mathbf{J}(\vec{r}_s) \approx \sum_{n=1}^{N} I_n \hat{f}_n(\vec{r}_s), \quad \hat{f}_n(\vec{r}_s) = \begin{cases} \frac{l_n}{2A^+_n} \tilde{\rho}^+_n, & \text{if } \vec{r}_s \in T^+_n \\ \frac{l_n}{2A^-_n} \tilde{\rho}^-_n, & \text{if } \vec{r}_s \in T^-_n \\ 0, & \text{if } \vec{r}_s \notin T_n \end{cases}
\]

Current basis function and testing function
MAS+Collocation Approach (simple 2D case)

\[ \{ r_n^0 \}_{n=1}^N, \quad r_n^0 \in S^0 \quad \{ r_m \}_{m=1}^M, \quad r_m \in S \]

\[ \sum_{n=1}^N \alpha_n Z_{nm} = V_m, \quad (m = 1, 2, \ldots, M) \]

\[ Z_{nm} = H_0^{(1)} \left( k \sqrt{(x_m - x_n^0)^2 + (y_m - y_n^0)^2} \right) \]

\[ V_m = E_z^{inc}(x_m, y_m) \]

Estimation of accuracy of solution

\[ \mathcal{E}^2 = \frac{\int_S \left| E_z^{inc}(x_s, y_s) - \sum_{n=1}^{N} \alpha_n H_0^{(1)} \left( k \sqrt{(x_s - x_n^0)^2 + (y_s - y_n^0)^2} \right) \right|^2 dS}{\int_S \left| E_z^{inc}(x_s, y_s) \right|^2 dS} ; \]
Method of Auxiliary Sources (problems)

Problem 1: unstability of linear system of algebraic equation

\[
\sum_{n=1}^{N} a_n \cdot Z_{nm} = V_m, (m = 1,2,\ldots, M)
\]

\[
\sum_{n=1}^{N} (a_n + \delta a_n) \cdot (Z_{nm} + \delta Z_{nm}) = V_m + \delta V_m
\]

Hadamard: for \( |\delta Z_{nm}| \approx 10^{-12} \) and \( |\delta V_m| \approx 10^{-12} \) \( \Rightarrow |\delta a_n| \approx a_n \)

Method of Auxiliary Sources (problems)

**Problem 2:** scattered fields main singularities (SFMS)

\[ \vec{E}^{\text{tot}} = \vec{E}^{\text{inc}} + \vec{E}^{\text{scat}} \]

From **Uniqueness Theorem**

The regular, in whole space, solution of Maxwell’s equation satisfying the radiation condition at infinity should be identically zero (\(\ast\)).

Scattered wave fields (both scalar and vector), which are continuously extended inside the scatterer’s domain certainly has irregular points (**singularities - SFMS**).

\[ \varphi(\vec{r}, \theta) = \frac{1}{4\pi\varepsilon_0} \left( \frac{q}{\sqrt{r^2 + (R + d)^2 - 2r(R + d) \cos \theta}} + \frac{q'}{\sqrt{r^2 + d^2 - 2rd \cos \theta}} \right) \]

\[ d = \frac{R^2}{R + h}; \quad q' = -\frac{Rq}{R + h} \]

V.D. Kupradze(\(\ast\)): “The main problems in the mathematical theory of diffraction”. GROL. Leningrad, Moscow 1935, 1-112.
**Method of Auxiliary Sources** (problems)

- **Problem 2:** scattered fields main singularities (SFMS - conformal mapping procedure)

- Image lines of primary source - \( L_1 \) and \( L_2 \);
- “Auxiliary Sources” (monopoles) surrounding the areas of SFMS concentration, imitating the radiation from the images \( L_1 \) and \( L_2 \);
- The lines of other possible distributions of auxiliary sources;
- Some optimal distribution of collocation points on the main surface \( S \).

Method of Auxiliary Sources (problems)

- **Problem 3**: Most general, linear form of constitutive relation (Bi-Isotropic medium)

\[
\tilde{D} = \varepsilon \tilde{E} + i \alpha \tilde{B}, \quad \tilde{H} = i \beta \tilde{E} + \mu^{-1} \tilde{B}
\]

**Isotropic magnetodielectrics**: \( \alpha = \beta = 0; \)

**Chiral medium**: \( \alpha = \beta \neq 0; \)

**Tellegen medium**: \( \alpha = -\beta \neq 0; \)

**Maxwell’s equations**

I. \( \tilde{\nabla} \times \tilde{E} = i \omega \tilde{B}; \) 

III. \( \tilde{\nabla} \times \tilde{H} = -i \omega \tilde{D}; \)

II. \( \tilde{\nabla} \cdot \tilde{B} = 0; \) 

IV. \( \tilde{\nabla} \cdot \tilde{D} = 0; \)

**Wave equation**

\[
\tilde{\nabla}^2 \tilde{U} + \kappa^2 \tilde{U} + \omega \mu (\alpha + \beta) \tilde{\nabla} \times \tilde{U} = 0, \quad \tilde{U} - \text{any field vectors}
\]

**Material parameters**

\[
\tilde{\varepsilon} = \begin{pmatrix} \varepsilon^r & \varepsilon^{r,\ell} \\ \varepsilon^{\ell,\ell} & \varepsilon^\ell \end{pmatrix}, \quad \varepsilon^{r,\ell} = \frac{k^{r,\ell}}{\omega \eta^{r,\ell}},
\]

\[
\tilde{\mu} = \begin{pmatrix} \mu^r & \mu^{r,\ell} \\ \mu^{\ell,\ell} & \mu^\ell \end{pmatrix}, \quad \mu^{r,\ell} = \frac{k^{r,\ell} \eta^{r,\ell}}{\omega}
\]

**Constitutive Relations**

**Wave impedances**

\[
\tilde{\eta} = \begin{pmatrix} \eta^r \\ \eta^\ell \end{pmatrix}, \quad \eta^{r,\ell} = \eta_0 \cdot \left( \sqrt{\eta^{-2} + \frac{(\alpha + \beta)^2}{4}} \mp \frac{\alpha - \beta}{2} \right)^{-1},
\]

\[
\eta_0 = \frac{\mu_0}{\sqrt{\varepsilon_0}}, \quad \eta = \sqrt{\frac{\mu_r}{\varepsilon_r}}
\]

**Wave numbers**

\[
\tilde{k} = \begin{pmatrix} k^r & 0 \\ 0 & -k^\ell \end{pmatrix}, \quad k^{r,\ell} = k \cdot \eta \cdot \left( \sqrt{\eta^{-2} + \frac{(\alpha + \beta)^2}{4}} \pm \frac{\alpha + \beta}{2} \right), \quad k = k_0 \sqrt{\varepsilon_r \mu_r}
\]

**Method of Auxiliary Sources** (problems)

- **Problem 3:** Fundamental Solution for Bi-Isotropic medium

\[
\Delta \times \vec{F} - \hat{k} \vec{F} = 0
\]

**Spinor basis**
\[
\vec{\tau} = \begin{pmatrix} \vec{\tau}^r \\ \vec{\tau}^\ell \end{pmatrix} = \begin{pmatrix} \vec{\tau}^r + i\vec{\tau}^u \\ \vec{\tau}^r - i\vec{\tau}^u \end{pmatrix}
\]

**Spinor of electromagnetic field**
\[
\vec{F} = \begin{pmatrix} \vec{F}^r \\ \vec{F}^\ell \end{pmatrix} = \begin{pmatrix} \vec{E} + i\mu^r \vec{H} \\ \vec{E} - i\mu^\ell \vec{H} \end{pmatrix}
\]

**Maxwell’s equation in the Majorana-Dirac form**
\[
\Delta \times \vec{F} - \hat{k} \vec{F} = 0
\]

**Fundamental Solution for Bi-Isotropic medium**
\[
\vec{F}_{n}^{r,\ell} = \nabla \times \left( \vec{G}_{n} \cdot \vec{\tau}_{n} \right) \pm \frac{1}{k^{r,\ell}} \nabla \nabla \left( \vec{G}_{n} \cdot \vec{\tau}_{n} \right) \pm \hat{k} \cdot \left( \vec{G}_{n} \cdot \vec{\tau}_{n} \right)
\]

**Green function matrix**
\[
\hat{G}_{n} = \begin{pmatrix} G_{n}^{r} & 0 \\ 0 & G_{n}^{\ell} \end{pmatrix}
\]

**Green function**
\[
G_{n}^{r,\ell} \left( \vec{p}, \vec{p}_{n} \right) = H_{0}^{(1)} \left( k^{r,\ell} \left| \vec{p} - \vec{p}_{n} \right| \right) - 2D case
\]
\[
G_{n}^{r,\ell} \left( \vec{r}, \vec{r}_{n} \right) = \frac{1}{4\pi} e^{\frac{i k^{r,\ell} \left| \vec{r} - \vec{r}_{n} \right|}{4\pi \left| \vec{r} - \vec{r}_{n} \right|}} - 3D case
\]

## General Concept of MAS

- Problem formulation for most general form of constitutive relations
- Geometry Analyzing for Scattered Field Main Singularities (SFMS)
- Definition of numerical method for evaluating the amplitudes of the auxiliary sources
- Deriving the system of linear algebraic equations
- Post processing stage

### Next Steps
- Definition of fundamental or other singular solutions of the wave equation
- Finding the best placement and type of the fundamental solutions (AS)
- Method of collocation, method of moments (MOM), orthogonalization
- Computation of amplitudes of AS
- Data processing and visualization
MAS Application to Some Particular Problems

- MAS approach for electromagnetic scattering on the Bi-Isotropic bodies;
- MAS simulation of wave propagation in Double-Negative medium;
- MAS and MMP simulations of Finite Photonic Crystal (PhC) based devices;
- MAS approach for electromagnetic scattering on Double-Periodic structures;
- MAS Approach for Band Structure Calculation and eigenvalue search problem;
Electromagnetic Scattering Upon the Chiral Bodies


Boundary problem for spinor field:
\[ \nabla \times \vec{F} - \hat{k} \vec{F} = 0 \]
\[ \hat{W} \vec{F}(\vec{r}) \bigg|_{\vec{r}=\vec{r}^s} = \vec{f}(\vec{r}^s), \quad M(\vec{r}^s) \in S \]

Required solution
\[ \vec{F}^{(N)}(\vec{r}) = \sum_{n=1}^{N} \hat{a}_n \vec{F}_n(\vec{r}, \vec{r}_n), \quad \vec{r} \in D, \quad \hat{a}_n = \begin{pmatrix} a^{r}_n & 0 \\ 0 & a^{\ell}_n \end{pmatrix} \]

Scattered electric field reconstructed from spinor fields
\[ \vec{E}^{sc}(\vec{r}) = \vec{E}^{r} + \vec{E}^{\ell} = \frac{\eta^r}{\eta^r + \eta^\ell} \sum_{n=1}^{N} a^{\ell}_n \vec{F}^{\ell}_n(\vec{r}, \vec{r}_n) + \frac{\eta^\ell}{\eta^r + \eta^\ell} \sum_{n=1}^{N} a^{r}_n \vec{F}^{r}_n(\vec{r}, \vec{r}_n) \]

Fundamental solution:
\[ \vec{F}^{r,\ell}_n = \nabla \times \left( \hat{G}_n \cdot \vec{\tau}_n \right) \pm \frac{1}{k^{r,\ell}} \nabla \nabla \left( \hat{G}_n \cdot \vec{\tau}_n \right) \pm \hat{k} \cdot \left( \hat{G}_n \cdot \vec{\tau}_n \right); \]
\[ \hat{G} = \begin{pmatrix} G^r & 0 \\ 0 & G^\ell \end{pmatrix}, \quad G^{r,\ell}(\vec{r} - \vec{r}') = \frac{1}{4\pi} \frac{e^{ik_n |\vec{r} - \vec{r}'|}}{|\vec{r} - \vec{r}'|}, \quad \vec{\tau}_n = \begin{pmatrix} \vec{\tau}^r_n \\ \vec{\tau}^\ell_n \end{pmatrix} = \begin{pmatrix} \vec{\tau}_v + i \vec{\tau}_u \\ \vec{\tau}_v - i \vec{\tau}_u \end{pmatrix} \]

\[ \hat{k} = \begin{pmatrix} k^r & 0 \\ 0 & -k^\ell \end{pmatrix} \]
\[ k^{r,\ell} = k \cdot \eta \left( \sqrt{\eta^2 + \frac{(\alpha + \beta)^2}{4}} \pm \frac{\alpha + \beta}{2} \right), \quad k = k_0 \sqrt{\varepsilon \mu} \]

Spherical shape chirolens

Constitutive relations
\[ \vec{D} = \varepsilon \vec{E} + i \alpha \vec{B}, \quad \vec{H} = i \beta \vec{E} + \mu^{-1} \vec{B} \]
\[ \alpha = \beta = \frac{0.3}{120\pi}, \quad \frac{\varepsilon}{\varepsilon_0} = 3.0, \quad \frac{\mu}{\mu_0} = 1.389, \quad k_0 = 400, \quad \text{height} - d = 0.6, \quad \text{thickness} - t = 0.125 \]
Plane Wave Excitation on Ciral Sphere

\[ a = 0.2 \text{ mm}; \varepsilon = \varepsilon_0 (5.0 + i0.1); \mu = (\mu_0 - \alpha^2 / \varepsilon)^{-1}; \alpha = \beta = \omega \varepsilon \gamma \]

Log\[\frac{\sigma}{\pi a^2}\]

Scattering cross-section versus frequency.

MAS approach: \( \gamma = 0; \gamma = 0.0001; \)

T-matrix approach: \( \gamma = 0.0001 \)

Kirchhoff-Kotler formula + MAS for Field Reconstruction in Double-Negative Medium (wave front reversal approach)

\[ k = k_0 \sqrt{-1} \cdot \sqrt{-1} \]

Gaussian beam illuminated transparent object
The total electric field \( E_z \) component reconstruction from space \( x<0 \) to space \( x>0 \). Total electric field \( E_z \) component was reconstructed from EM field tangential components on the YOZ plane.

Electric field distribution from a Gaussian beam source left \( x<0 \) (actual) and MAS predicted electric field distribution in a virtual DNM half space right \( x>0 \) (reconstructed from EM field tangential components on YOZ plane).

David Karkashadze, Juan Pablo Fernandez, and Fridon Shubitidze: “Scatterer localization using a left-handed medium”. OPTICS EXPRESS 9906, (C) 2009 OSA, 8 June 2009 / Vol. 17, No. 12
MAS and Multiple Multipole Method (MMP) Simulations of Finite Photonic Crystal (PhC) Based Devices


“Filtering T-junction” Design (MMP, MAS)

Comparison (without optimization)

<table>
<thead>
<tr>
<th>Case</th>
<th>MMP simulation</th>
<th>MAS simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R (%)</td>
<td>T_L (%)</td>
</tr>
<tr>
<td>Left</td>
<td>35.37</td>
<td>63.38</td>
</tr>
<tr>
<td>Right</td>
<td>36.51</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Results of Filtering T_Junction optimization

- \( f_1 = 1.038 \times 10^{14} \text{Hz} \)
- \( f_2 = 1.230 \times 10^{14} \text{Hz} \)
- \( R_{up} = 0.73\% \)
- \( R_{up} = 0.76\% \)
- \( T_{right} = 0.16\% \)
- \( T_{right} = 97.77\% \)
- \( T_{left} = 98.59\% \)
- \( T_{left} = 0.88\% \)
Coupling a Slab with a PhC Waveguide: $fa/c=0.38$ (MMP, MAS)

Before optimization:
- $SWR_{WG_{MAS}} = 1.72$
- $SWR_{WG_{MMP}} = 1.66$

After optimization:
- $SWR_{WG} = 1.08$

$D_{WG} = 3.379$; $h_{PhC_{WG}} = 1.644$
3D Double-Periodic Green Functions

\[ \Pi(x, y, z) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \exp(iknd_x \cos \theta_x + ikmd_y \cos \theta_y) \frac{\exp\left(ik\sqrt{(x-nd_x)^2 + (y-nd_y)^2 + z^2}\right)}{ik\sqrt{(x-nd_x)^2 + (y-nd_y)^2 + z^2}} \]

Poisson Transformation

\[ \Pi(x, y, z) = -\frac{2\pi}{d_x d_y} \sum_{p=-\infty}^{+\infty} \sum_{q=-\infty}^{+\infty} \frac{1}{ik_{zp}} \exp(ik_{xq} x + ik_{yp} y + ik_{zp} z) \]

\[ k_{xq} = k \cos \theta_x + \frac{2\pi}{d_x} q, \quad k_{yp} = k \cos \theta_y + \frac{2\pi}{d_y} p, \quad h_p = \sqrt{k^2 - k_{yp}^2}, \quad p = 0, \pm 1, \pm 2, \ldots, q = 0, \pm 1, \pm 2, \ldots \]

\[ k_{zp} = \begin{cases} \sqrt{h_p^2 - k_{xq}^2}, & \text{if } h_p > k_{xq} \text{ and } k > k_{yp} \\ i\sqrt{k_{xq}^2 - h_p^2}, & \text{if } h_p < k_{xq} \text{ and } k > k_{yp} \end{cases} \Rightarrow k_{zp} = i\sqrt{h_p^2 + k_{xq}^2}, \text{ if } k < k_{yp} \]

Oblique Incident of Plane Wave on Bi-Periodic Array of Dielectric Spheres

a) Problem geometry (radius of spheres - $a$, period – $d$, incident angle $\theta=20^\circ$, $\varphi=0^\circ$, permittivity of dielectric spheres $\varepsilon=3.0$, $a/d=0.4$);

b) Transmission coefficient versus relative period. The solid curve is for p-polarization and the dotted curve is for s-polarization. Comparison of MAS approach with results presented in

Dielectric layer with hexagonally bi-periodic dents: a) problem geometry and incidents plane wave orientation; b) transmission coefficient versus relative period.
MAS and MMP Approach for Band Structure Calculation

2D bi-periodic structure with arbitrary shaped dielectric scatterers and unit cell with distribution of auxiliary sources (left);
Periodic Green’s functions for different angles of incidence a) $\theta=0\degree$, b) $\theta=45\degree$ (right).

Band structure for a perfect PhC made of dielectric rods (left); Band structure for a perfect PhC made of silver wires (right).

Conclusion

The procedure of solution of scattering problems by the method of auxiliary sources (MAS) needs preliminary considerations of various problems.

Correct solution of these problems can radically influence efficiency of MAS.
Thank you for attention

david.karkashadze@emcos.ge