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Modeling of Optical Waveguides with Material and Radiation Loss

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  - Geometrical arrangement
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Introduction
Optical Waveguides with Material and Radiation Losses

Measured $\varepsilon_r$ of silver:

Complex $\varepsilon_r \rightarrow$ material losses

Open problem $\rightarrow$ material losses
Introduction
MMP and FEM Eigenvalue Analysis

Multiple Multipole Program (MMP)
- Boundary method
- no air box and ABCs are required
- Frequency domain
- Large overdetermined dense linear systems of equations
- “Fictitious” excitation required
- Eigenvalue searching / tracing functions are of paramount importance

Finite Element Method (FEM)
- Domain method
- Air box and its truncation (ABCs) are required
- Frequency domain
- Large square sparse linear systems of equations
- Direct extraction of eigenvalues and eigenvectors
- Initial guess of a certain eigenvalue is required
Introduction

MMP Eigenvalue Analysis

Fictitious excitation (last expansion)

\[
\text{Residual} = \frac{\text{Ax} - b}{\text{Amplitude}}
\]

Wavelength \( \lambda \) is fixed.
Propagation constant \( \gamma \) is an eigenvalue.

Search function in the complex \( \gamma \) plane:

Even mode
Odd mode

\[ \lambda = 500\text{nm} \]

Fun.1 = Residual / Amplitude
Introduction
MMP Eigenvalue Analysis

Search functions:
(1) Residual/Amplitude, (2) Residual, (3) 1/Amplitude
(4) Relative Error Average, (5) Relative Error Maximum,
(6) Absolute Error Average / Field Average,
Introduction

FEM Eigenvalue Analysis

Wavelength $\lambda$ is fixed (Dispersive materials and ABCs).

Propagation constant $\gamma$ is an eigenvalue.

Hybrid modes are expected!

\[
\nabla \times \left( \frac{1}{\varepsilon_r} \nabla \times \vec{H} \right) - \mu_r k_0^2 \vec{H} = 0, \quad \vec{H}(x, y, z) = \vec{H}(x, y) \cdot e^{-jk_z z}
\]

\[
\vec{n} \times \left( \nabla \times \vec{H} \right) - \left( jk + \frac{1}{2r} \right) \vec{n} \times (\vec{H} \times \vec{n}) = 0
\]

Vector triangular elements must be applied!

FEM discretization yields a generalized nonlinear eigenvalue problem:

\[
[A(\gamma)] \cdot x = 0
\]

Triangular mesh
Introduction
MMP and FEM Eigenvalue Analysis

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Optical Fiber with Metallic Cladding

Geometric Arrangement

Glass

$\varepsilon_r = 2.25$

50nm

300nm

Air

AG

AG

AG

AG
Optical Fiber with Metallic Cladding
MMP Modeling

Complex $y$ plane:

$$\lambda=350\text{nm}, \text{ Function 3}$$

$$f(e) = \frac{\text{Error}(e)}{\text{Amplitude}(e)}$$
Optical Fiber with Metallic Cladding

MMP vs. FEM

365nm |E|

365nm |E|

390nm

340nm

371nm

348nm

340nm

365nm

365nm

371nm

348nm

434nm

588nm

Rea(\text{gamma})/k_0

Imag(\text{gamma})/k_0
Optical Fiber with Metallic Cladding
MMP vs. FEM
Optical Fiber with Metallic Cladding
MMP vs. FEM

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Rudimentary Dielectric Photonic Crystal Waveguide

Geometric Arrangement

x – propagation direction

TE modes: \( \vec{E} = E_z \hat{e}_z \)

BVP:

\[
\frac{\partial}{\partial x} \left( \frac{1}{\mu} \frac{\partial E_z}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\mu} \frac{\partial E_z}{\partial y} \right) + \omega^2 \varepsilon_0 \varepsilon_z E_z = 0 , \text{ in the computational domain}
\]

\[
\frac{\partial E_z}{\partial y} + jk_0 E_z = 0 , \text{ ABC over the absorbing boundary (red)}
\]

\[
E_z(x_2,y) = E_z(x_1,y) \cdot e^{-j\gamma(x_2-x_1)} , \text{ Floquet periodicity (green)}
\]

\[
\frac{\partial E_z}{\partial n} = \frac{\partial E_z}{\partial y} = 0 , \text{ Symmetry (Perfect Magnetic Conductor)}
\]
Rudimentary Dielectric Photonic Crystal Waveguide
MMP Modeling

Expansions:

- Dielectric rod: Bessel expansion
- Upper half plane: Rayleigh expansion
- Periodic cell: Multipole expansions
- Periodic boundary condition:

\[ \text{Field}(x + dx, y, z) = \text{Field}(x, y, z) \cdot e^{jC_x \cdot dx} \]
Function 3 in the complex $C_x$ space, $\lambda=2.7\mu m$

$$f(e) = \frac{\text{Error}(e)}{\text{Amplitude}(e)}$$
Rudimentary Dielectric Photonic Crystal Waveguide
MMP vs. FEM

Function 3 in the complex $C_x$ space, $\lambda=2.7\mu$m

$$f(e) = \frac{\text{Error}(e)}{\text{Amplitude}(e)}$$
Rudimentary Metallic Photonic Crystal Waveguide
MMP Modeling

Function 3 in the complex $C_x$ space

$$f(e) = \frac{\text{Error}(e)}{\text{Amplitude}(e)}$$
Conclusions

- Eigenmodes of the waveguides with radiation and material losses have very complicated traces in the complex propagation constant plane.
- FEM eigenvalue analysis needs a good initial guess which is for these waveguides not so simple.
- MMP analysis improves our understanding of the behaviour of guiding modes and reveals all relevant eigenvalues.
- FEM is very fast for eigenvalue tracing over a large wavelength range as the eigenvalue found before is usually an accurate guess for the next step.
- MMP eigenvalue trace over a large wavelength range is radically accelerated by using the Parameter Estimation Technique (PET).
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